PHYSICS 320: Midterm Exam
Wed. Oct. 7 2009
10 AM - 12 PM

Each problem carries 10 points. You are allowed to refer to your notes from class and tutorials and also the graded homework problems but no other material. Attempt all questions.

1. Consider the jellium model for electrons of density n in d dimensions. Assume that the interaction between two electrons continues to have the form $V(r) = e^2/r$ as a function of the distance r between them. The ground state energy per unit volume in the Hartree-Fock approximation is given by

$$E = \frac{1}{V_d} \sum_{\mathbf{p}} \left[\frac{p^2}{2m} + \frac{1}{2} \epsilon_{ex}(\mathbf{p}) \right],$$

where $\epsilon_{ex}(\mathbf{p})$ is the exchange energy of an electron of momentum $\mathbf{p}$ and V_d is the d dimensional volume of the system. In the parts below, you do not need to calculate the constants A, B and C.

   (a) $r_s = An^{\alpha}$. What is α? (*2 points*)

   (b) Show that the kinetic energy contribution to E is of the form $\frac{1}{V_d} \sum_{\mathbf{p}} \frac{p^2}{2m} = B/r_s^{\beta}$. What is β? (*4 points*)

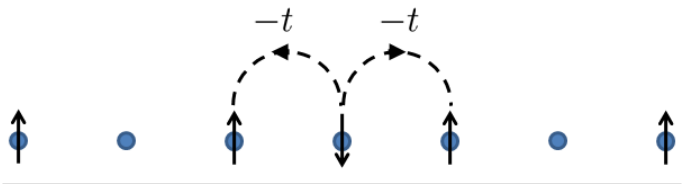
   (c) Show that the exchange contribution to E is of the form $\frac{1}{V_d} \sum_{\mathbf{p}} \frac{1}{2} \epsilon_{ex}(\mathbf{p}) = -C/r_s^{\gamma}$. What is γ? (*4 points*)

2. For a 1D gas of free fermions of density n,

   (a) Show that there is a continuum of particle-hole excitations about the ground state. Sketch this continuum in the ω and q plane, where $\hbar\omega$ is the energy of an excitation and $\hbar q$ its momentum, and derive expressions for its boundaries. (*6 points*)

   (b) Focus on the part of the spectrum with $|q| \ll k_F$. Calculate the range of excitation energies $\Delta(q)$ and the average energy $E_{av}(q)$ for a given q. Show that the ratio $\Delta(q)/E_{av}(q) \to 0$ as $q \to 0$. (*4 points*).

3. Consider a one dimensional lattice of spacing a with up and down spin electrons. The up spin electrons are immobile and there is one on every alternate site. The down spin electrons can hop between nearest neighbour sites with amplitude $-t$. There is a Coulomb repulsion energy U if there are two electrons on the same site.



   (a) Write down an expression for the Hamiltonian of this system. (*4 points*)

   (b) Calculate the band structure of the down spin electrons assuming periodic boundary conditions. Make rough sketches of the band(s). (*6 points*)