PHYSICS 320: Assignment No. 8
Due: November 30, 2009

1. Consider a thin metallic film of thickness a. The mean free path l_0 in a bulk sample under the same conditions is greater than a. Show that if the electrons hitting the surface are scattered randomly, the electrical conductivity σ of the thin film parallel to its surface is related to that of the bulk metal (σ_0) by

$$\sigma/\sigma_0 = 3a/4l_0 + \frac{a}{2l_0}\ln(l_0/a).$$

2. Calculate the resistivity of a liquid metal, assuming that the ionic potential seen by an electron can be approximated by a weak local pseudopotential and the scattering cross section can be calculated using the Born approximation. Show that the resistivity is given by

$$\rho = \frac{3\pi m}{8\hbar e^2}\frac{V}{E_f}\int_0^{2k_f} S(q)|V_s(q)|^2\frac{q^3}{k_f^4}dq\,,$$

where V is the volume, $S(q)$ is the static structure factor of the liquid and $V_s(q)$ is the Fourier transform of the pseudopotential.

3. Using the BCS gap equation, show that the temperature-dependence of the gap parameter Δ at low temperatures ($T \ll T_c$) is given by

$$\Delta(T) \simeq \Delta_0 - (2\pi\Delta_0 T)^{1/2}e^{-\Delta_0/T}\,,$$

where $\Delta_0 \equiv \Delta(T=0)$.

4. Consider a model of superconductivity in which the effective interaction between electrons has the form

$$V(\mathbf{k}_1,\mathbf{k}_2) = -v_1\Theta(\omega_D - |\xi(\mathbf{k}_1)|)\Theta(\omega_D - |\xi(\mathbf{k}_2)|) + v_2\Theta(\Omega_p - |\xi(\mathbf{k}_1)|)\Theta(\Omega_p - |\xi(\mathbf{k}_2)|)$$

with v_1, $v_2 > 0$, $\Omega_p \gg \omega_D$ and $\xi(\mathbf{k}) = E(\mathbf{k}) - \mu$. Assume that the gap function $\Delta(\mathbf{k})$ is given by

$$\Delta(\mathbf{k}) = \Delta_1 \text{ if } |\xi(\mathbf{k})| < \omega_D,\ \Delta(\mathbf{k}) = \Delta_2 \text{ if } \omega_D < |\xi(\mathbf{k})| < \Omega_p,$$

and $\Delta(\mathbf{k}) = 0$ if $|\xi(\mathbf{k})| > \Omega_p$. Show that in the weak-coupling limit, $\Delta(\mathbf{k}) \ll \omega_D$, the gap parameter Δ_1 at $T = 0$ satisfies the equation

$$N'(0)v_{eff}\ln(2\omega_D/\Delta_1) = 1\,,$$

where

$$v_{eff} = v_1 - v_2[1 + N'(0)v_2\ln(\Omega_p/\omega_D)]^{-1}.$$