PHYSICS 320: Assignment No. 7
Due: November 16, 2009

1. The lifetime of a phonon is infinite in the harmonic approximation, but it becomes finite when anharmonic effects are taken into account. Considering only the first anharmonic term in the phonon Hamiltonian, list the processes that make the lifetime of a phonon finite. Which of these processes is the most important one at very low temperatures?

   Assume that the dispersion of acoustic phonons in a system is given by $\omega(k) = ck + \alpha k^2$ in the long wavelength limit ($c$ is the speed of sound and $\alpha$ is a constant). Using arguments based on the conservation of energy and crystal momentum, show that the lifetime of a long-wavelength acoustic phonon in this system diverges at very low temperatures if $\alpha < 0$.

2. Ashcroft and Mermin, Chapter 26, problem no. 1.

3. Ashcroft and Mermin, Chapter 26, problem no. 2.

4. The Hamiltonian of the combined electron-phonon system can be solved exactly if the electrons are assumed to be *fixed* in their positions. Assume that there are $n$ electrons fixed at positions $\{\mathbf{r}_i\}$, $i = 1, 2, \ldots, n$ (e.g. localized electrons in deep core levels or impurity levels). Obtain exact expressions for the energy eigenvalues of the electron-phonon system. Show that the phonons induce an effective interaction between pairs of electrons.