PH 206: Electromagnetic theory

Exam # 3: Mon. Feb 8 2013

Total points: 30 **Time: 2 hrs.**

You are not allowed to consult any written, printed or electronic material. Attempt all questions. All the best.

1. In this problem you will consider the magnetostatics of an infinitely long solenoid filled with a linear magnetic material of permeability μ. The axis of the solenoid is along $\hat{z}$, its radius is R and it has a wire carrying a current I wound around it with N turns per unit length. (**10 points**)

   (a) Calculate $\mathbf{B}$, $\mathbf{H}$ and $\mathbf{M}$ everywhere. (3 points)

   (b) Now, imagine that a hole of radius $a < R$ is bored coaxially in the magnetic material. Argue that the fields $\mathbf{B}$, $\mathbf{H}$ and $\mathbf{M}$ are everywhere along $\hat{z}$ and calculate their values. (4 points)

   (c) Now, suppose that instead of boring a coaxial hole, the magnetic material is cut perpendicular to its axis and the two halves separated by a small distance. Are the fields the fields $\mathbf{B}$, $\mathbf{H}$ and $\mathbf{M}$ everywhere along $\hat{z}$? Why? You do not need to calculate the values of the fields. (3 points)

2. Recall the expression for the Maxwell Stress Tensor

$$T_{\alpha\beta} = \epsilon_0 \left(E_\alpha E_\beta - \frac{1}{2}\delta_{\alpha\beta} E^2 \right) + \frac{1}{\mu_0} \left(B_\alpha B_\beta - \frac{1}{2}\delta_{\alpha\beta} B^2 \right),$$

   where E_α and B_α are components of the electric and magnetic fields respectively. (**10 points**)

   (a) Consider a conducting sphere of radius R carrying charge Q. Use the above expression for the stress tensor to obtain the force on a patch of area dA on the surface of the sphere. Verify that this is the same force you get if you calculate it directly from electrostatics. (5 points)

   (b) Now consider an infinitely long cylinder of radius R with axis along $\hat{z}$. A uniform current of density $K\hat{\phi}$ per unit length flows on the surface. Use the expression for the stress tensor to obtain the force on a patch of area of dA on the surface of the cylinder. Verify that this is the same force you get directly from magnetostatics. (5 points)

3. Consider a localized static charge distribution which produces an electric field $\mathbf{E} = -\boldsymbol{\nabla}\phi$ and a localized static current distribution $\mathbf{J}(\mathbf{r})$ generating a magnetic field $\mathbf{B}$. (**10 points**)

   (a) Show that the momentum of the electromagnetic fields is given by

$$\mathbf{P} = \frac{1}{c^2} \int \phi \mathbf{J} d^3\mathbf{r},$$

   if the product $\phi\mathbf{B}$ falls of sufficiently fast at infinity and the integral is over all space. How fast is sufficiently fast? (5 points)

   (b) Now, suppose that $\mathbf{E}$ varies so slowly that it can be assumed to be essentially constant over the spatial extent of $\mathbf{J}$. Show that in this case, the momentum

$$\mathbf{P} = \frac{1}{c^2}\mathbf{E} \times \mathbf{m},$$

   where $\mathbf{m}$ is the magnetic moment of the current distribution. (*Hint: Recall that for a static distribution of current* $\int \mathbf{J}\ d^3\mathbf{r} = 0$ *and* $\int x_i J_j d^3\mathbf{r} = -\int x_j J_i d^3\mathbf{r}$.)(5 points)