PH 206: Electromagnetic theory

Problem Set 6

1. This problem is the magnetostatic analogue of problem # 4 from PS #4.
   (a) A sphere of radius R made of a magnetic material of permeability μ is placed in a uniform magnetic field $\mathbf{B_0}$. Calculate $\mathbf{B}$, $\mathbf{H}$ and $\mathbf{M}$ everywhere in space. What is the total induced dipole moment in the sphere and the bound volume and surface current densities?
   (b) Now, assume there is no external field but the sphere has a frozen in magnetization $\mathbf{M}$, which gives the same dipole moment as in the previous part. Calculate $\mathbf{B}$, $\mathbf{H}$ and $\mathbf{M}$ everywhere in space and the bound currents.

2. A superconductor develops a magnetization when placed in a magnetic field just like any other material. However, it has the special property that the bound current density $\mathbf{J_b} = \alpha\mathbf{A}$, where $\mathbf{A}$ is the magnetic vector potential and α is a positive constant.
   (a) What is the differential equation that the magnetic field $\mathbf{B}$ obeys at any point inside the conductor?
   (b) Now assume that all of space for $z < 0$ is filled with a superconductor. Assume that there is a magnetic field in the region $z > 0$ such that $B_z = 0$ and the field has no dependence on x and y. Show that the magnetic field decays as you go deeper and deeper into the superconductor. What is the field at a point at depth d below $z = 0$ in terms of $\mathbf{B_0}$, the field at $z = 0$? What is the characteristic length scale over which the magnetic field decays?
   (c) The differential equation in (a) also admits a solution in which the magnetic field increases as you go deeper into the superconductor. Why would you discard this solution on physical grounds?

3. Consider a perfectly conducting rod of mass m which is free to slide without friction if allowed between conducting rails separated by a distance d. There is a battery of voltage V connected through a resistor of resistance R across the rails. Further there is also a constant magnetic field B perpendicular to the rails and the rod. Neglect the self inductance of the circuit.
   (a) Assume that the rod is initially fixed in place and suddenly released at time $t = 0$. Calculate the current in the circuit and the speed of the rod as functions of t.
   (b) Show that the power supplied by the battery is equal to the sum of the power dissipated by the resistor and the rate of change of kinetic energy of the rod.
   (c) Assume now that a (possibly velocity dependent) force $F(v)$ acts on the rod along its line of motion. Show that the power supplied by the battery equals the sum of the power dissipated by the resistor, the rate of change of kinetic energy of the rod and the rate of work done by the force. Thus depending on the sign of the force, extra energy per unit time cane be extracted from or delivered to the battery compared to when the force is zero.
   (d) Repeat parts (a) and (b) with a current source I instead of battery V. The rest of the system remains unchanged.

4. This problem is the magnetostatic analogue of problem #5 from PS #4. An infinitely long solenoid is connected to a current source that supplies a constant current. This results in a magnetizing field $\mathbf{H}_0$ inside the solenoid. A magnetic material of volume V in the shape of a solid cylinder is placed inside the solenoid parallel to its axis so that it develops a uniform magnetization density $\mathbf{M}$. Show that the work done by the current source to change the magnetization by an amount $d\mathbf{M}$ is $dW = -\mu_0\mathbf{H}_0.d\mathbf{M}V$.

5. Consider two current loops 1 and 2 of arbitrary shape of self inductances L_{11} and L_{22} respectively. Identify a point O_1 on loop 1 and O_2 on loop 2 and say that the displacement vector between them is $\mathbf{R}$. The mutual inductance L_{12} is then a function of $\mathbf{R}$.
   (a) What is the energy $E(\mathbf{R})$ in terms of the above quantities?
   (b) The force between the two loops $\mathbf{F}(\mathbf{R})$ is also a function of $\mathbf{R}$. Show that $\boldsymbol{\nabla}_\mathbf{R}.\mathbf{F}(\mathbf{R}) = 0$ and as a result $\nabla^2_\mathbf{R} L_{12}(\mathbf{R}) = 0$.