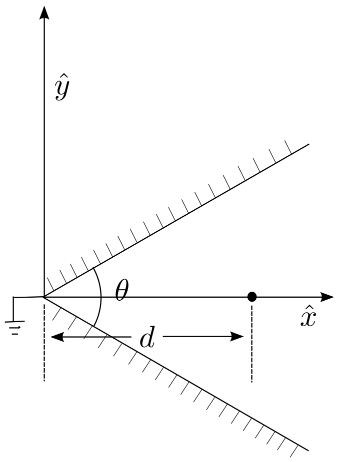
# PH 206: Electromagnetic theory

**Midterm exam: Wed. Feb 27 2013**

**Total points: 30** **Time: 2 hrs.**

**You are not allowed to consult any written, printed or electronic material. Attempt all questions. All the best.**

1. Two semi-infinite grounded conducting planes making an angle $\theta$ meet along the $z$ axis. The lines of intersection of the two planes with the $xy$ plane are bisected by the $x$ axis as shown in the figure below. A charge $q$ is placed on the $x$ axis at a distance $d$ from the origin. (**10 points**)




   (a) If the potential in the region between the two planes can be calculated using $n$ image charges, argue that $n$ is odd. What is the value of the angle $\theta$ given $n$? (3 points)

   (b) What are the charges and positions of the image charges for a given $n$? Express the positions in cylindrical polar coordinates $r$, $\phi$ and $z$. What is the total induced charge on each grounded plane? (4 points)

   (c) What are the appropriate limits to take for $n$ and $d$ so that the problem reduces to that of a point charge $q$ equidistant from two parallel infinite conducting planes separated by a distance $d'$? (3 points)

2. Consider an infinite right circular cylinder of radius $R$ made of a uniform linear dielectric material of permittivity $\epsilon$. Let its axis be along $\hat{z}$. The quantities $\mathbf{E}$, $\mathbf{P}$, $\mathbf{D}$ and $\rho_b$ have their usual meanings and $\sigma_b$ is the charge density on the curved surface of the cylinder. (**10 points**)

   (a) Calculate $\mathbf{E}$, $\mathbf{P}$, $\mathbf{D}$ inside and outside the cylinder when it is placed in a previously uniform field $E_0\hat{z}$. Also calculate $\rho_b$ and $\sigma_b$. (3 points)

   (b) Now, calculate the same quantities for a previously uniform field $E_0\hat{x}$. (7 points)

   It might help to perform the calculation in terms of cylindrical polar co-ordinates $\rho$, $\phi$ and $z$. The gradient, divergence and Laplacian in terms of these coordinates are

$$\begin{aligned} \boldsymbol{\nabla} f &= \frac{\partial f}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z} \\ \boldsymbol{\nabla}.\mathbf{V} &= \frac{1}{\rho}\frac{\partial(\rho V_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial V_\phi}{\partial \phi} + \frac{\partial V_z}{\partial z} \\ \nabla^2 f &= \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}. \end{aligned}$$

   Also, remember that the general solution to Laplace's equation $\nabla^2 f = 0$, when $f$ has no dependence on $z$ is

$$f(\rho,\phi) = \sum_{n=0}^{\infty}\left(A_n\rho^n + B_n\rho^{-n}\right)\left(C_n\cos n\phi + D_n\sin n\phi\right),$$

where $n$ is an integer.

3. Calculate the magnetic field at all points in space due to each of the following: (**10 points**)

   (a) A very thin wire carrying current $I$ along the $z$ axis from $z = \infty$ to $z = -\infty$. (*This is not a trick question!*) (2 points)

   (b) A uniform current per unit length $K\hat{x}$ flowing in the $xy$ plane. (3 points)

   (c) A very thin wire carrying current $I$ along the $z$ axis from $z = \infty$ to the origin. At the origin, the current spreads out uniformly and radially along the $xy$ plane. (5 points).