PH 206: Electromagnetic theory

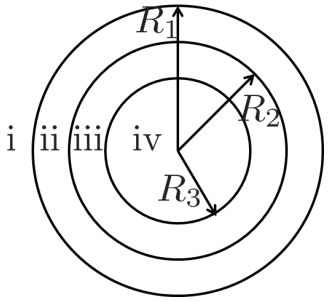
Exam # 1: Fri. Feb 1 2013

Total points: 30 **Time: 2 hrs.**

You are not allowed to consult any written, printed or electronic material. Attempt all questions. All the best.

1. Consider two metallic regions of capacitances C_1 and C_2 bounded by surface $\partial\Omega_1$ and $\partial\Omega_2$ respectively. It can be shown that if $\partial\Omega_1$ can be completely contained inside $\partial\Omega_2$, $C_1 < C_2$. Use this fact to find upper and lower bounds for the capacitance of a metallic cube of side a. (5 points)

2. Consider three concentric hollow metallic spheres of radii R_1, R_2 and R_3 as shown in the figure below. The outermost sphere has charge q_1, the middle one q_2 and the inner one q_3. (15 points)



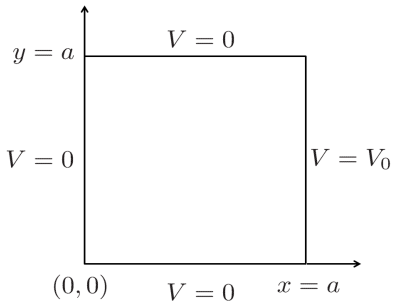
   (a) Calculate the potential as a function of r, the distance from the origin in each of the four regions indicated: i ($r \geq R_1$), ii ($R_1 > r \geq R_2$), iii ($R_2 > r \geq R_3$), iv ($r < R_3$). (6 points)

   (b) Calculate the total electrostatic energy stored in the system. (5 points)

   (c) Suppose the innermost and outermost spheres are connected by a very thin metallic wire. What are the charges on the three spheres now? (4 points)

3. A square of side length a has its four sides maintained at constant potentials whose values are shown in the figure below. The potential $V(x, y)$ inside the square obeys Laplace's equation. (10 points)
   Recall that the general solution to Laplace's equation in cartesian coordinates in 2D is

$$V(x,y) = \sum_{k\neq 0} \left(\alpha_k e^{kx}\cos ky + \beta_k e^{-kx}\cos ky + \gamma_k e^{kx}\sin ky + \delta_k e^{-kx}\sin ky\right) + \epsilon xy + \eta x + \rho y + \xi.$$



   (a) Calculate $V(x, y)$ and the electric field $\mathbf{E}(x, y)$ everywhere inside the square. You can leave your answers as infinite series. (7 points)

   (b) Now, suppose the that the side which was at potential V_0 is grounded and the three sides which were grounded are maintained at potential V_0. How is $V(x, y)$ in this case related to what you obtained in the previous part? (*Hint: The relation is very simple and does not require any elaborate calculation.*) (3 points)