PH 206: Electromagnetic theory

Final Exam: Sat. May 4 2013

Total points: 30 **Time: 3 hrs.**

You are not allowed to consult any written, printed or electronic material. Attempt all questions. All the best.

1. In this problem you will consider the passage of polarized light through something called a Hall insulator. RCP (LCP) refers to right (left) circularly polarized light which is defined such that if the fingers of the right (left) hand curl in the direction of rotation of the electric field as a function of time at any point on a light beam, the thumb points along the direction of its propagation. The conductivity tensor is defined by the relation

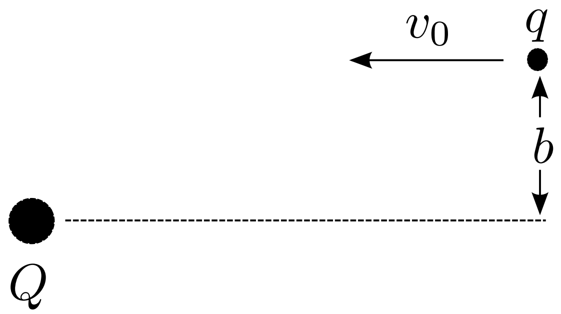
$$J_\alpha = \sigma_{\alpha\beta} E_\beta,$$

where $\mathbf{J}$ is the electrical current density and $\mathbf{E}$, the electric field. The Hall insulator has the following components of $\sigma_{\alpha\beta}$: $\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \sigma_{xz} = \sigma_{zx} = \sigma_{yz} = \sigma_{zy} = 0$ and $\sigma_{xy} = -\sigma_{yx} = \sigma_H$, where $\sigma_H > 0$ is a constant. Consider the propagation of light waves of frequency ω in the $\hat{z}$ direction.(**10 points**)

   (a) Starting from Maxwell's equations, show that you obtain a set of coupled linear equations for E_x and E_y, which can be written as a 2×2 matrix equation. Show that the eigenvectors of this matrix correspond to RCP and LCP, which are thus the only two kinds of polarized light which propagate without any change in polarization. (5 points)

   (b) What are the values of the wavevectors k for RCP and LCP? Show that there is a threshold frequency ω_c below which one of the two modes will not propagate. What is the value of ω_c and is it RCP or LCP which does not propagate for $\omega < \omega_c$? (*Hint: Compute the eigenvalues of the matrix of the previous part.*) (5 points)

2. A particle of mass m and charge q moves from infinity towards a fixed charge Q with impact parameter b and initial speed v_0 as shown in the figure. (**10 points**)



   (a) What are the conditions that have to be satisfied by the above parameters so that the motion of the particle is non-relativistic and essentially along a straight line? (4 points)

   (b) Assume that the conditions of the above part are satisfied and that the particle moves non-relativistically and essentially along a straight line. What is the total energy radiated by the particle? Recall that the Larmor formula for the power radiated by an accelerated charge is

$$P = \frac{1}{4\pi\epsilon_0} \frac{2}{3} \frac{q^2 a^2}{c^3},$$

   where a is the magnitude of the instantaneous acceleration. (6 points)

3. In the unprimed frame O, the entire xy plane carries a uniform surface current density $\mathbf{K} = \kappa\hat{x}$, where κ is a constant. The charge density is zero everywhere in O. The primed frame O' moves with uniform velocity $v\hat{z}$ relative to O. The axes of the two frames coincide at $t = t' = 0$. (**10 points**)

   (a) What are the electric field $\mathbf{E}(\mathbf{r}, t)$ and magnetic field $\mathbf{B}(\mathbf{r}, t)$ in O? (2 points)

(b) What are the electric field $\mathbf{E}'(\mathbf{r}', t')$, magnetic field $\mathbf{B}'(\mathbf{r}', t')$, charge density $\rho'(\mathbf{r}', t')$ and current density $\mathbf{J}'(\mathbf{r}', t')$ in O'? *The electric and magnetic fields in this part have to expressed as functions of* $\mathbf{r}'$ *and* t' *and not* $\mathbf{r}$ *and* t. *Further the current density* $\mathbf{J}'$ *is the volume current density, i.e. current per unit area and not surface current density.* (4 points)

(c) Show that $\mathbf{E}'(\mathbf{r}', t')$ and $\mathbf{B}'(\mathbf{r}', t')$ obey Maxwell's equations in O' by explicitly taking derivatives with respect to $\mathbf{r}'$ and t'. (4 points).

Useful formulae

$$\int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \pi$$

$$\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^2} = \frac{\pi}{2}$$

$$\int_{-\infty}^{\infty} \frac{dx}{1+x^4} = \frac{\pi}{\sqrt{2}}$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{pmatrix}$$

Lorentz transformation between frames O and O' of problem #3.

$$x' = x,$$

$$y' = y,$$

$$z' = \gamma\left(z - vt\right),$$

$$t' = \gamma\left(t - \frac{v}{c^2} z\right),$$

where $\gamma = 1/\sqrt{1 - v^2/c^2}$.

Sign function

$$\mathrm{Sgn}(x) = 1 \text{ for } x > 0,\ \mathrm{Sgn}(x) = -1 \text{ for } x < 0$$

Delta function

$$\delta(x) = \frac{1}{2}\frac{d\mathrm{Sgn}(x)}{dx}$$

$$\delta(ax) = \frac{1}{|a|}\delta(x)$$