Polarization

$$\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t+\varphi)}$$

We know that $\vec{E} \perp \hat{k}$, so more generally

$$\vec{E} = \left(E_1 e^{i\varphi_1}\hat{\varepsilon}_1 + E_2 e^{i\varphi_2}\hat{\varepsilon}_2\right) e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

or $\vec{E} = \left[E_1 \hat{\varepsilon}_1 + E_2 e^{i\Delta\varphi}\hat{\varepsilon}_2\right] e^{i(\vec{k}\cdot\vec{r}-\omega t+\varphi_1)}$

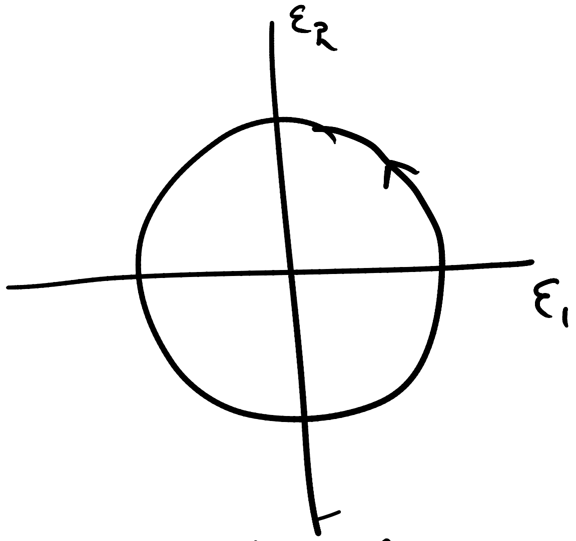
If $\Delta\varphi = 0$

$\vec{E}$ is linearly polarized, the electric field is always in the direction $\dfrac{E_1\hat{\varepsilon}_1 + E_2\hat{\varepsilon}_2}{\sqrt{E_1^2+E_2^2}}$

Angle wrt $\hat{\varepsilon}_1$ direction is $\tan\theta = \dfrac{E_2}{E_1}$

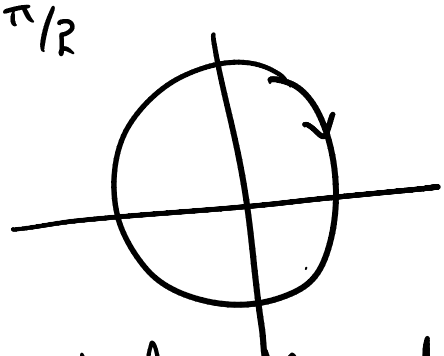
Suppose $E_1 = E_2 = E$ and $\Delta\varphi = \dfrac{\pi}{2}$

$$\mathrm{Re}(\vec{E}) = E\cos(\vec{k}\cdot\vec{r}-\omega t)\hat{\varepsilon}_1 - E\sin(\vec{k}\cdot\vec{r}-\omega t)\hat{\varepsilon}_2$$



Left circularly polarized light
positive helicity

$\Delta\varphi = -\pi/2$



Right circularly polarized, negative helicity

New basis vectors

$$\hat{\varepsilon}_{\pm} = \frac{1}{\sqrt{2}}\left(\hat{\varepsilon}_1 \pm i\,\hat{\varepsilon}_2\right)$$

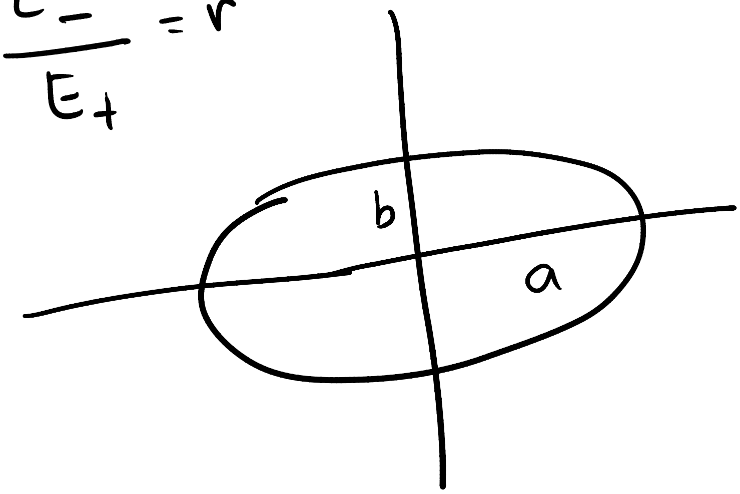
$$\hat{\varepsilon}_{\pm}^{*} \cdot \hat{\varepsilon}_{\pm} = 1 \; ; \quad \hat{\varepsilon}_{\pm}^{*} \cdot \hat{\varepsilon}_{\mp} = 0$$

$\vec{E}(\vec{r},t) = \left(E_+ \hat{\varepsilon}_+ + E_- \hat{\varepsilon}_-\right) e^{i(\vec{k}\cdot\vec{r} - \omega t + \varphi)}$ 3

Elliptically polarized light

E_+ and E_- have no phase difference

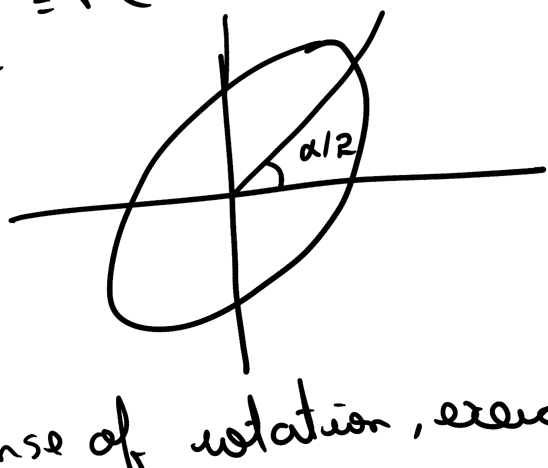
$\frac{E_-}{E_+} = r$



$\frac{a}{b} = \left|\frac{1+r}{1-r}\right|$

With a phase difference α

$\frac{E_-}{E_+} = re^{i\alpha}$



Sense of rotation, exercise

Stokes parameters

$\hat{\varepsilon}_1 \cdot \vec{E}$, $\hat{\varepsilon}_2 \cdot \vec{E}$, $\hat{\varepsilon}_+^* \cdot \vec{E}$ and $\hat{\varepsilon}_-^* \cdot \vec{E}$

$E_1 = a_1 e^{i\delta_1}$, $E_2 = a_2 e^{i\delta_2}$, $E_+ = a_+ e^{i\delta_+}$,

$E_- = a_- e^{i\delta_-}$

$$S_0 = |\hat{\varepsilon}_1 \cdot \vec{E}|^2 + |\hat{\varepsilon}_2 \cdot \vec{E}|^2 = a_1^2 + a_2^2$$

$$S_1 = |\hat{\varepsilon}_1 \cdot \vec{E}|^2 - |\hat{\varepsilon}_2 \cdot \vec{E}|^2 = a_1^2 - a_2^2$$

$$S_2 = 2 \,\mathrm{Re}\left[(\hat{\varepsilon}_1 \cdot \vec{E})^* (\hat{\varepsilon}_2 \cdot \vec{E}) \right] = 2 a_1 a_2 \cos(\delta_1 - \delta_2)$$

$$S_3 = 2 \,\mathrm{Im}\left[(\hat{\varepsilon}_1 \cdot \vec{E})^* (\hat{\varepsilon}_2 \cdot \vec{E}) \right] = 2 a_1 a_2 \sin(\delta_1 - \delta_2)$$

Exercise: Write Stokes parameters in terms of $\hat{\varepsilon}_\pm$ & $\vec{E}_\pm$

$S_0^2 = S_1^2 + S_2^2 + S_3^2$; Only 3 independent real parameters required to specify state of polarization

If light is not perfectly monochromatic, for instance unpolarized

$S_\alpha = \langle S_\alpha \rangle$, $\langle \ \rangle$ average over time $\gg \frac{1}{\omega}$.

In general $S_0^2 \geq S_1^2 + S_2^2 + S_3^2$

For completely unpolarized light

$S_1 = S_2 = S_3 = 0$