Lorentz force

$$\vec{F} = q\,\vec{v} \times \vec{B}$$

$$\Rightarrow \vec{F} = \int_{\Omega} d\vec{r}\,(\vec{J} \times \vec{B})$$

In 1D

$$\vec{F} = \int dl\,(\vec{I} \times \vec{B})$$

Force on a dipole

$$\vec{B}(\vec{r}) = \vec{B}(0) + (\vec{r}\cdot\vec{\nabla})\vec{B}(0) + \cdots$$

$$\vec{F} = \int_{\Omega} d\vec{r}'\left[\vec{J} \times \left\{\vec{B}(0) + (\vec{r}'\cdot\vec{\nabla}')\vec{B}(0) + \cdots\right\}\right]$$

$$\int_{\Omega} \vec{J}\,d\vec{r}' = 0 \quad (\because \vec{\nabla}\cdot\vec{J} = 0$$
$$\text{\& } \vec{J} \text{ is localized})$$

$$\vec{F} = \int_{\Omega} d\vec{r}'\,\vec{J}\,(\vec{r}'\cdot\vec{\nabla}')\vec{B}(0)$$

$$= \iint_{\Omega\,\Omega} d\vec{r}\,d\vec{r}'\,\vec{J}(\vec{r}'\cdot\vec{\nabla})\vec{B}(\vec{r})\delta(\vec{r})$$

$$\vec{J}(\vec{r}')\left[\vec{r}'\cdot\vec{\nabla}\right]\vec{B}(\vec{r}')$$

$$= \vec{J}(\vec{r}')\times\vec{\nabla}\left[\vec{r}'\cdot\vec{B}(\vec{r})\right] \quad (\because \vec{\nabla}\times\vec{B}=0$$

for the external field)

$$= -\vec{\nabla}\times\left[\vec{J}(\vec{r}')\,\vec{r}'\cdot\vec{B}(\vec{r})\right]$$

$$\vec{F} = -\vec{\nabla}\times\int \vec{J}(\vec{r}')\left[\vec{r}'\cdot\vec{B}(\vec{r})\right]d\vec{r}'\Big|_{\vec{r}=0}$$

$$= \vec{\nabla}\times(\vec{B}\times\vec{m})\Big|_{\vec{r}=0}$$

$$= (\vec{m}\cdot\vec{\nabla})\vec{B}\Big|_{\vec{r}=0} \quad (\because \vec{\nabla}\cdot\vec{B}=0)$$

$$= \vec{\nabla}(\vec{m}\cdot\vec{B})\Big|_{\vec{r}=0} \quad (\because \vec{\nabla}\times\vec{B}=0)$$

Total torque

$$\vec{\tau} = \int \vec{r}\times(\vec{J}\times\vec{B})\,d\vec{r}$$

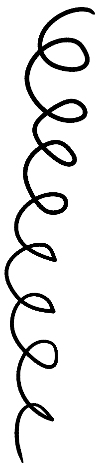
$$\vec{\tau} = \vec{m}\times\vec{B}(0)$$

Potential energy in an external field $U = -\vec{m}\cdot\vec{B}$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

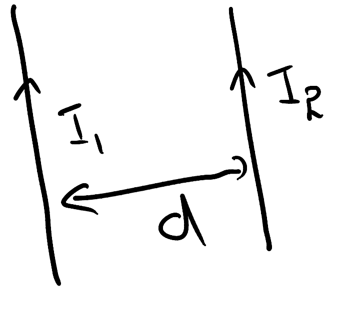
$$\Rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 \vec{I}$$

$\Rightarrow$



$\vec{B} = \mu_0 n I$; n - # of turns

$$\vec{B} = \frac{\mu_0 I}{2\pi r} \hat{\theta}$$



$$f = -\frac{\mu_0 I_1 I_2}{2\pi d}$$

f - force per unit length

Magnet materials

$$\vec{A} = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$\vec{A} = \frac{\mu_0}{4\pi} \int_{\Omega} \frac{\vec{m}(\vec{r}') \times \hat{\mathscr{r}}}{\mathscr{r}^2} d\vec{r}'$$

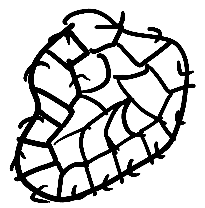
$$= \frac{\mu_0}{4\pi} \int_{\Omega} \left[\vec{m}(\vec{r}') \times \vec{\nabla}' \frac{1}{\mathscr{r}}\right] d\vec{r}'$$

$$= \frac{\mu_0}{4\pi} \left[\int_{\Omega} \frac{1}{\mathscr{r}} [\vec{\nabla}' \times \vec{m}(\vec{r}')] d\vec{r}' - \int_{\Omega} \vec{\nabla}' \times \left(\frac{\vec{m}}{\mathscr{r}}\right) d\vec{r}'\right]$$

$$= \frac{\mu_0}{4\pi} \int_{\Omega} \frac{1}{\mathscr{r}} \vec{\nabla}' \times \vec{m} \, d\vec{r}' + \frac{\mu_0}{4\pi} \int_{\partial\Omega} \frac{\vec{m} \times d\vec{S}'}{\mathscr{r}}$$

$$\vec{J}_b = \vec{\nabla} \times \vec{m} \; ; \; \vec{K}_b = \vec{m} \times \hat{n}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}_b}{\mathscr{r}} d\vec{r}' + \frac{\mu_0}{4\pi} \int_{\partial\Omega} \frac{\vec{K}_b}{\mathscr{r}} dS'$$



Current loops induced

Surface current flows. If loops don't exactly cancel in the bulk, bulk current density.

Magnetostatic fields in matter

$$\vec{J} = \vec{J}_f + \vec{J}_b$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

$$\vec{\nabla} \times \vec{m} = \vec{J}_b$$

$$\vec{\nabla} \times \left(\frac{\vec{B}}{\mu_0} - \vec{m} \right) = \vec{J}_f = \vec{\nabla} \times \vec{H}$$

$$\vec{B} = \mu_0 (\vec{H} + \vec{m})$$

$$\oint \vec{H} \cdot d\vec{l} = I_f$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \; ; \; \vec{\nabla} \times \vec{m} = \vec{J}_b \; ; \; \vec{\nabla} \times \vec{H} = \vec{J}_f$$

$$\vec{\nabla} \cdot \vec{B} = 0 \; ; \; \vec{\nabla} \cdot \vec{m} = -\vec{\nabla} \cdot \vec{H}$$

Boundary conditions

$$B_{\perp}^{out} = B_{\perp}^{in}$$

$$\vec{H}_{\parallel}^{out} - \vec{H}_{\parallel}^{in} = \vec{K}_f \times \hat{n}$$

$$\vec{B}_{\parallel}^{out} - \vec{B}_{\parallel}^{in} = \vec{K} \times \hat{n}$$

$$\vec{M}_{\parallel}^{out} - \vec{M}_{\parallel}^{in} = \vec{K}_b \times \hat{n}$$

$$H_{\perp}^{out} - H_{\perp}^{in} = M_{\perp}^{in} - M_{\perp}^{out}$$

Linear magnetic materials

$$\vec{B} = \mu \vec{H}$$

$$\vec{M} = \chi \vec{H}$$

$\mu = \mu_0(1+\chi)$; μ - permeability

χ - susceptibility

$\chi > 0$ paramagnets or ferromagnets

$\chi < 0$ diamagnets ($\chi \geq -1$)

$\chi = -1$ perfect diamagnet (superconductor)