Magnetic materials

Work done = Work done only to set up fields currents.

$$\delta W = \int \vec{H} \cdot \delta \vec{B} \, d\vec{r}$$

analogous to

$$\delta W = \int \vec{E} \cdot \delta \vec{D} \, d\vec{r}$$

Energy of a linear magnetic material in an external magnetic field $\vec{B}_0$

$$W = \frac{1}{2} \int \vec{m} \cdot \vec{B}_0 \, d\vec{r}$$

Coefficients of inductance

$$\vec{J} \propto \vec{A} \Rightarrow W = \frac{1}{2} \int \vec{J} \cdot \vec{A} \, d\vec{r}$$

$$W = \frac{1}{2} \sum_{i=1}^{N} L_i I_i^2 + \frac{1}{2} \sum_{i \neq j} M_{ij} I_i I_j$$

L_i - self inductance

M_{ij} - mutual inductance

$$W = \frac{\mu_0}{8\pi} \int d\vec{r} \int d\vec{r}' \; \frac{\vec{J}(\vec{r}) \cdot \vec{J}(\vec{r}')}{|\vec{r} - \vec{r}'|}$$

$$L_i = \frac{\mu_0}{8\pi I_i^2} \int d\vec{r} \int d\vec{r}' \; \frac{\vec{J}_i(\vec{r}) \cdot \vec{J}_i(\vec{r}')}{|\vec{r} - \vec{r}'|}$$

$$M_{ij} = \frac{\mu_0}{8\pi I_i I_j} \int d\vec{r} \int d\vec{r}' \; \frac{\vec{J}_i(\vec{r}) \cdot \vec{J}_j(\vec{r}')}{|\vec{r} - \vec{r}'|}$$

$$M_{ij} = M_{ji}$$