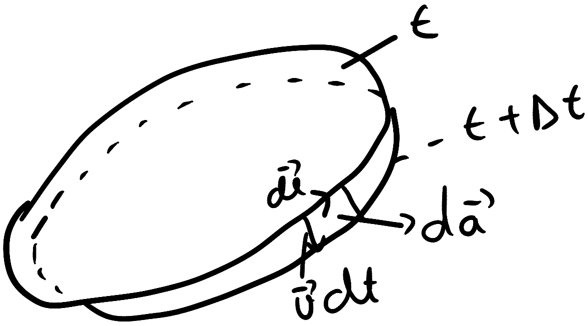
Faraday's Law

Lorentz force

$$\vec{F} = q(\vec{v} \times \vec{B}) = q\,\vec{E}_{ind}$$

$$\vec{E}_{ind} = \vec{v} \times \vec{B}$$



$$\oint \vec{E}_{ind} \cdot d\vec{l} = \int (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

$$d\vec{a} = -\,d\vec{l} \times \vec{v}\,dt$$

$$\oint \vec{E}_{ind} \cdot d\vec{l} = \iint_{\Delta S} (d\vec{l} \times \vec{v}) \cdot \vec{B} = -\frac{1}{\Delta t} \iint_{\Delta S} \vec{B} \cdot d\vec{a}$$

ΔS – area of the strip -ween the two loops

$$\oint \vec{E}_{ind} \cdot d\vec{l} = -\frac{\Delta \Phi_s}{\Delta t}$$

$$\oiint_{\text{entire surface}} \vec{B} \cdot d\vec{S} = 0 \qquad \Delta \Phi_s + \Phi(t) - \Phi(t+\Delta t) = 0$$

$$\Rightarrow \oint \vec{E}_{ind} \cdot d\vec{l} = -\frac{d\Phi}{dt}$$

$$\oint \vec{E}_{ind} \cdot d\vec{l} = -\frac{d\varphi}{dt} = \varepsilon$$ (Faraday's law / Lenz's law)

$$\varphi = \int_{\text{surface}} \vec{B} \cdot d\vec{S}$$

$$\frac{d\varphi}{dt} = \int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} + \oint (\vec{B} \times \vec{v}) \cdot d\vec{l}$$

$$\Rightarrow \oint \left[\vec{E}_{ind} - (\vec{v} \times \vec{B}) \right] \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

Frame of reference in which loop is instantaneously at rest $\vec{v} = 0$ & $\oint \vec{E} \cdot d\vec{l} = -\iint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$

(Galilean invariance)

$$\vec{E}_{ind} = \vec{E} + \vec{v} \times \vec{B}$$

$$\oint \vec{E} \cdot d\vec{l} = -\iint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

$$\iint \left(\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} \right) \cdot d\vec{S} = 0$$ for all surfaces

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Galilean invariance

$$\vec{E}' = \vec{E} + \vec{v} \times \vec{B}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

In the general case

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \text{ and } \vec{E}' \neq \vec{E} + \vec{v} \times \vec{B}$$

Equality holds only for $v \ll c$

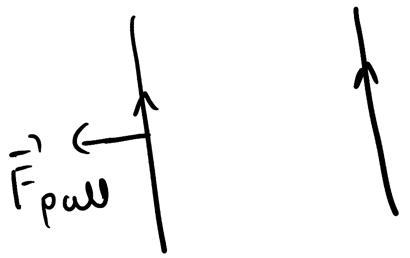
Magnetic work

$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$\vec{F} \cdot d\vec{r} = 0 \quad \because (\vec{v} \times \vec{B}) \cdot d\vec{r} = 0$$

$$\vec{v} = \frac{d\vec{r}}{dt}$$

Work done by other forces



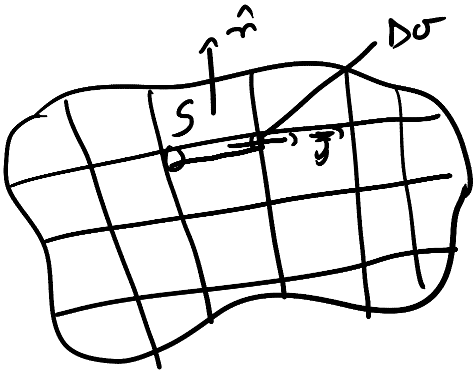
$$\vec{F}_{pull} + \vec{F}_{batt} + \vec{F}_{mag} = 0$$

$$\Rightarrow \int \vec{F}_{pull} \cdot d\vec{r} = -\int \vec{F}_{batt} \cdot d\vec{r} \quad \left(\because \int \vec{F}_{mag} \cdot d\vec{r} = 0\right)$$

$$d\omega_{pull} = -d\omega_{batt}$$

$$\frac{d\omega_{batt}}{dt} = \mathcal{E} I = -I\frac{d\varphi}{dt}$$

$$d\omega_{pull} = I\, d\varphi = d\omega$$



$$\Delta(\delta\omega) = J\,\Delta\sigma \int_S \delta\vec{B}\cdot\hat{n}\, d\vec{s}$$

$$\delta\vec{B} = \vec{\nabla}\times\delta\vec{A}$$

$$\Delta(\delta\omega) = J\,\Delta\sigma \int_S (\vec{\nabla}\times\delta\vec{A})\cdot\hat{n}\, d\vec{s}$$

$$= J\,\Delta\sigma \oint_C \delta\vec{A}\cdot d\vec{l}$$

$$d\vec{l} \parallel \vec{J}$$

$$\delta\omega = \int(\delta\vec{A}\cdot\vec{J})\, d\vec{r} \quad ; \delta\omega = \sum \Delta(\delta\omega)$$

$$\vec{\nabla}\times\vec{H} = \vec{J}$$

$$\delta \omega = \int \delta \vec{A} \cdot (\vec{\nabla} \times \vec{H}) d\vec{r}$$

$$\delta \omega = \int \vec{H} \cdot (\vec{\nabla} \times \delta \vec{A}) d\vec{r}$$

$$+ \int \vec{\nabla} \cdot (\vec{H} \times \delta \vec{A}) d\vec{r}$$

If $\vec{J}$ is localized and $\vec{A} \to 0$ as $r \to \infty$.

$$\delta \omega = \int \vec{H} \cdot \delta \vec{B} d\vec{r}$$

Linear magnetic material

$$\omega = \frac{1}{2} \int \vec{H} \cdot \vec{B} d\vec{r}$$

In free space

$$\omega = \frac{1}{2\mu_0} \int B^2 d\vec{r}$$

$$\vec{A} \propto \vec{J} \Rightarrow \omega = \frac{1}{2} \int \vec{J} \cdot \vec{A} d\vec{r}$$