# WORK AND ENERGY



$$\vec{E} = -\vec{\nabla} V$$

$$\vec{F} = q\vec{E} = -q\vec{\nabla} V$$

Work- Energy Theorem

Potential energy $\mathcal{E}_{AB} = -\int_A^B \vec{F} \cdot d\vec{l}$

$$= q \int_A^B \vec{\nabla} V \cdot dl = q (V_B - V_A)$$

If A is at $r = \infty$ so that $V_A = 0$

$$\mathcal{E} = qV$$

A COLLECTION OF POINT CHARGES

$$PE_i = q_i \left( \frac{1}{4\pi\varepsilon_0} \sum_{j \neq i} \frac{q_j}{|\vec{r}_i - \vec{r}_j|} \right)$$

$$\mathcal{E} = \sum_i PE_i = \frac{1}{2} \frac{1}{4\pi\varepsilon_0} \sum_{\substack{i,j \\ j \neq i}} \frac{q_i q_j}{|\vec{r}_i - \vec{r}_j|}$$

to account for double counting

$$\mathcal{E} = \frac{1}{2} \sum_{\substack{i,j \\ i \neq j}} q_i V_j$$

Continuous charge distributions

$$\mathcal{E} = \frac{1}{2} \int_{\Omega} \rho V \, d\vec{r}$$

$\rho$ is contained in $\Omega$

$$= \frac{\varepsilon_0}{2} \int_{\Omega} (\vec{\nabla} \cdot \vec{E}) V \, d\vec{r}$$

$$= \frac{\varepsilon_0}{2} \left[ \int_{\Omega} \vec{\nabla} \cdot (V\vec{E}) \, d\vec{r} - \int_{\Omega} \vec{E} \cdot \vec{\nabla} V \, d\vec{r} \right]$$

$$= \frac{\varepsilon_0}{2} \int_{\partial\Omega} V \vec{E} \cdot d\vec{S} + \frac{\varepsilon_0}{2} \int_{\Omega} E^2 \, d\vec{r}$$

Take $\Omega = \mathbb{R}^3$ $\quad V\vec{E} \sim \frac{1}{r^3}$ on $\partial\Omega$

$dS \sim r^2$ so $\displaystyle\int_{\partial\Omega} V\vec{E} \cdot dS \sim \frac{1}{r}$

So $\mathcal{E} = \frac{\varepsilon_0}{2} \int_{\mathbb{R}^3} \vec{E}^2 \, d\vec{r}$

Potential energy of a charge distribution $\rho$,

$$\mathcal{E} = \frac{\varepsilon_0}{2} \int \vec{E}^2 \, d\vec{r}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$