Electromagnetic waves

| 0 sources

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E}$$

$$= -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\Rightarrow \nabla^2 \vec{E} = \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} \quad \text{where } c^2 = \frac{1}{\mu_0 \varepsilon_0}$$

IIIly $\nabla^2 \vec{B} = \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}$

1D wave eqn

$$\frac{\partial^2 \vec{E}}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}$$

General solution

$$\vec{E} = \vec{E}_1 (x - ct) + \vec{E}_2 (x + ct)$$

Wave solutions

$\vec{E} = \vec{E}_0 e^{i[k(x-ct)+\phi]}$ or $\vec{E} = \vec{E}_0 e^{i[k(x+ct)+\phi]}$

Speed = c   speed = -c

$\vec{E} = \vec{E}_0 e^{i(kx-\omega t+\phi)}$ $\vec{E} = \vec{E}_0 e^{i[kx+\omega t+\phi]}$

Forward moving wave   Backward moving wave

In 3D

$\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t+\phi)}$ Plane monochromatic wave

$\vec{\nabla}\cdot\vec{E} = 0 \Rightarrow \vec{k}\cdot\vec{E} = 0$ or $\vec{k} \perp \vec{E}$

$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow i\vec{k}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\Rightarrow \vec{B} = \vec{B}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t+\phi)}$

where $\vec{B}_0 = \frac{1}{c}\hat{k}\times\vec{E}_0$

$\vec{E}$, $\vec{B}$ and $\vec{k}$ are $\perp$ to each other

Plane EM waves are transverse

The actual fields are real parts of $\vec{E}$ and $\vec{B}$

$$\vec{E} = \tilde{\vec{E}}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)} \quad \& \quad \vec{B} = \tilde{\vec{B}}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$\tilde{\vec{E}}_0$ and $\tilde{\vec{B}}$ include the phases $e^{i\phi}$

Energy in EM wave

$$\varepsilon = \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) \quad ; \text{ Energy/unit volume}$$

$$= \varepsilon_0 E^2 \cos^2(\vec{k}\cdot\vec{r}-\omega t+\phi)$$

$$\langle \varepsilon \rangle = \frac{1}{2}\varepsilon_0 E^2$$

$$\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$$

$$= c\,\varepsilon_0 E^2 \cos^2(\vec{k}\cdot\vec{r}-\omega t+\phi)\,\hat{k}$$

$$\langle \vec{S} \rangle = \frac{1}{2} c\,\varepsilon_0 E^2 \hat{k}$$

$$\vec{P} = \frac{1}{c}\varepsilon_0 E^2 \cos^2(\vec{k}\cdot\vec{r}-\omega t+\phi)\,\hat{k}$$

$$\langle \vec{P} \rangle = \frac{1}{2c}\varepsilon_0 E^2 \hat{k}$$

Power / unit area $= \langle S \rangle = \frac{1}{2} c\,\varepsilon_0 E^2 = I$ (intensity)

$\langle \mathcal{E} \rangle = \frac{I}{c}$

Pressure = radiation pressure = $|\langle \vec{S} \rangle| c$

$= \frac{I}{c}$

Polarization

$\vec{E} = \vec{\tilde{E}}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$

$\vec{\tilde{E}}_0 \perp \vec{k}$

Let $\vec{k} \parallel \hat{z}$

$\vec{\tilde{E}}_0 = \tilde{E}_x \hat{x} + \tilde{E}_y \hat{y}$

$\tilde{E}_x = E_x e^{i\varphi_x}$; $\tilde{E}_y = E_y e^{i\varphi_y}$

$\Delta\varphi = \varphi_x - \varphi_y$

1 $\varphi = 0 \Rightarrow$ linearly polarized light