EM waves in media (1)

Linear medium

$$\vec{D} = \varepsilon \vec{E} \quad ; \quad \vec{B} = \mu \vec{H}$$

No free charges or currents

$$\vec{\nabla} \cdot \vec{D} = 0 \quad \Rightarrow \quad \vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \quad \Rightarrow \quad \vec{\nabla} \times \vec{B} = \mu \varepsilon \frac{\partial \vec{E}}{\partial t}$$

$$\nabla^2 \vec{E} = \mu \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$v = \frac{1}{\sqrt{\mu \varepsilon}} \qquad \frac{v}{c} = \sqrt{\frac{\mu_0 \varepsilon_0}{\mu \varepsilon}}$$

In many cases of interest $\mu = \mu_0$

so $\frac{v}{c} = \sqrt{\frac{\varepsilon_0}{\varepsilon}} = \frac{1}{n}$

where n is the refractive index

Energy density $= \frac{1}{2} \varepsilon E^2 + \frac{1}{2\mu} B^2$

Poynting vector $\vec{S} = \frac{1}{\mu} (\vec{E} \times \vec{B})$

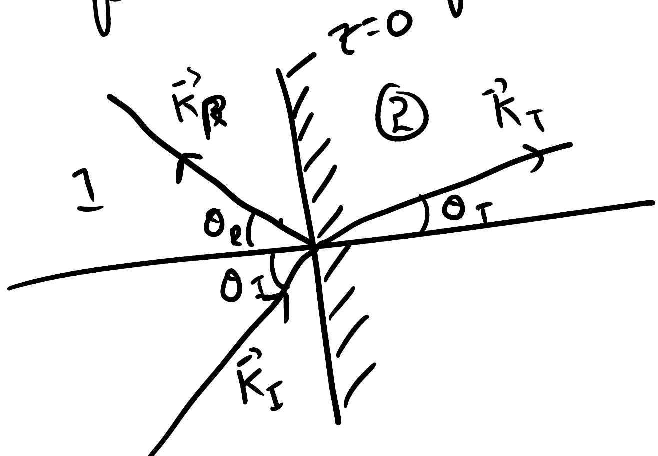
$I = \frac{1}{2} \varepsilon v E_0^2$ - intensity

Boundary conditions

$\varepsilon_1 E_\perp^1 = \varepsilon_2 E_\perp^2$; $\vec{E}_{\parallel}^1 = \vec{E}_{\parallel}^2$

$B_\perp^1 = B_\perp^2$; $\frac{\vec{B}_{\parallel}^1}{\mu_1} = \frac{\vec{B}_{\parallel}^2}{\mu_2}$

Reflection & Refraction



$\vec{E}_I = \vec{E}_{0I} e^{i(\vec{k}_I \cdot \vec{r} - \omega_I t)}$; $\vec{B}_I = \frac{1}{v_1}(\hat{k}_I \times \vec{E}_I)$

$\vec{E}_R = \vec{E}_{0R} e^{i(\vec{k}_R \cdot \vec{r} - \omega_R t)}$; $\vec{B}_R = \frac{1}{v_1}(\hat{k}_R \times \vec{E}_R)$

$\vec{E}_T = \vec{E}_{0T} e^{i(\vec{k}_T \cdot \vec{r} - \omega_T t)}$; $\vec{B}_T = \frac{1}{v_2}(\hat{k}_R \times \vec{E}_T)$

Fields in 1 $\vec{E}_I + \vec{E}_R$; $\vec{B}_I + \vec{B}_R$

Fields in 2 $\vec{E}_T$; $\vec{B}_T$

BC's of the form

$$(\ \ ) e^{i(\vec{k}_I \cdot \vec{r} - \omega_I t)} + (\ \ ) e^{i(\vec{k}_R \cdot \vec{r} - \omega_R t)} = (\ \ ) e^{i(\vec{k}_T \cdot \vec{r} - \omega_T t)} \quad \forall t$$

O

$\Rightarrow \omega_I = \omega_R = \omega_T = \omega$

$\Rightarrow v_1 k_I = v_1 k_R = v_2 k_T$

or $k_I = k_R = \frac{v_2}{v_1} k_T$

BC also true for all x & y at $z=0$

$\Rightarrow x(k_I)_x + y(k_I)_y = x(k_R)_x + y(k_R)_y$

$= x(k_T)_x + y(k_T)_y \quad \forall (x,y)$

$\Rightarrow (k_I)_x = (k_R)_x = (k_T)_x$

and $(k_I)_y = (k_R)_y = (k_T)_y$

Let $\vec{k}_I$ lie in the xz plane so $(k_I)_y = 0$

$\Rightarrow (k_R)_y = (k_T)_y = 0$ or $\vec{k}_T$ and $\vec{k}_R$

also lie in the xz plane

$$\sin\theta_I = \frac{\vec{k}_I \cdot \hat{x}}{k_I} = \frac{(k_I)_x}{k_I}$$

$$= \frac{(k_R)_x}{k_R} = \sin\theta_R \Rightarrow \theta_I = \theta_R$$

$$\sin\theta_T = \frac{\vec{k}_T \cdot \hat{x}}{k_T} = \frac{(k_T)_x}{k_T}$$

$$= \frac{v_2}{v_1} \frac{(k_I)_x}{k_I} = \frac{v_2}{v_1} \sin\theta_I$$

$$\frac{\sin\theta_T}{\sin\theta_I} = \frac{v_2}{v_1} = \frac{n_1}{n_2}$$ Snell's law

$$\varepsilon_1\left[E_{0I}^z + E_{0R}^z\right] = \varepsilon_2 E_{0T}^z$$

$$B_{0I}^z + B_{0R}^z = B_{0T}^z$$

$$E_{0I}^{(x,y)} + E_{0R}^{(x,y)} = E_{0T}^{(x,y)}$$

$$\frac{B_{0I}^{(x,y)} + B_{0R}^{(x,y)}}{\mu_1} = \frac{B_{0T}^{(x,y)}}{\mu_2}$$

Assume that $\vec{E}_I$ is in xz plane

$\Rightarrow$ $\vec{E}_R$ and $\vec{E}_T$ are also in the xz plane

$$\varepsilon_1\left(-E_{0I}\sin\theta_I + E_{0R}\sin\theta_I\right) = -\varepsilon_2 E_{0T}\sin\theta_T$$

$$E_{0I}\cos\theta_I + E_{0R}\cos\theta_R = E_{0T}\cos\theta_T$$

$$\frac{E_{0I} - E_{0R}}{\mu_1} = \frac{E_{0T}}{\mu_2}$$

$$\beta = \frac{\mu_1}{\mu_2}\frac{n_1}{n_2}; \quad \alpha = \frac{\cos\theta_T}{\cos\theta_I}$$

$$E_{0R} = \frac{\alpha - \beta}{\alpha + \beta} E_{0I} \; ; \; E_{0T} = \frac{2}{\alpha + \beta} E_{0I}$$

Fresnel equations

$\alpha = \beta \Rightarrow$ reflected wave extinguished, all the light is transmitted

Angle $\sin^2 \theta_B = \dfrac{1 - \beta^2}{\left(\frac{n_1}{n_2}\right)^2 - \beta^2}$

θ_B - Brewster angle

$\theta_I > \theta_B$, no reflected wave

No such angle for polarization $\perp$ to the plane.

Thus in general for $\theta_I > \theta_B$, reflected wave is polarized $\perp$ to the plane.