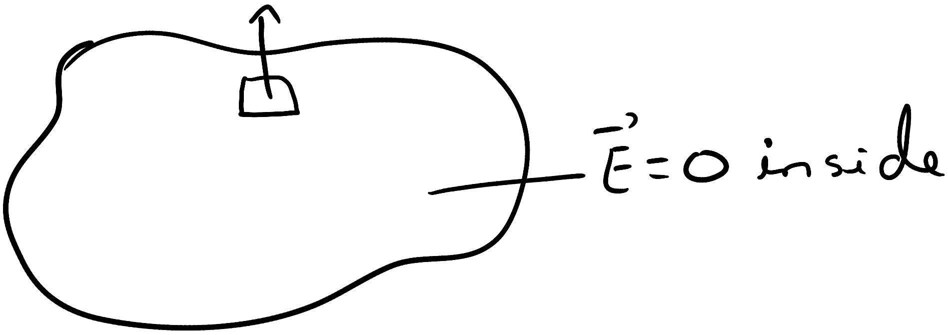
Force on a conductor



$$\vec{E}_{tot} = 0 \text{ inside}$$
$$= \frac{\sigma}{\varepsilon_0}\hat{n} \text{ outside}$$

$$\vec{E}_{tot} = \vec{E}_{rest} + \vec{E}_{patch} = \frac{\sigma}{\varepsilon_0}\hat{n} \text{ outside}$$
$$= \vec{E}_{rest} - \vec{E}_{patch} = 0 \text{ inside}$$

$$\Rightarrow \vec{E}_{patch} = \frac{\sigma}{2\varepsilon_0}\hat{n} = \vec{E}_{rest}$$

Force on patch $\vec{F} = \delta q\, \vec{E}_{rest} = \dfrac{\delta q\, \sigma}{2\varepsilon_0}\hat{n}$

Pressure $P = \dfrac{\sigma^2}{2\varepsilon_0}$ (Electrostatic pressure)

Capacitors - two conductors

charge $+Q$ and $-Q$ (usually so

field $\neq 0$ only in confined region)

$$Q = C_{11} V_1 + C_{12} V_2 \qquad (C_{12} = C_{21})$$

$$-Q = C_{12} V_1 + C_{22} V_2$$

$$C_{11} V_1 + C_{12} V_2 = -C_{12} V_1 - C_{22} V_2$$

$$V_2 = -\frac{(C_{11} + C_{12}) V_1}{C_{12} + C_{22}}$$

$$Q = V_1 \left[C_{11} - C_{12} \frac{(C_{11} + C_{12})}{C_{12} + C_{22}} \right]$$

$$= V_1 \left[\frac{C_{11} C_{22} - C_{12}^2}{C_{12} + C_{22}} \right]$$

$$V_1 - V_2 = \left(\frac{C_{11} + C_{22} + 2C_{12}}{C_{12} + C_{22}} \right) V_1$$

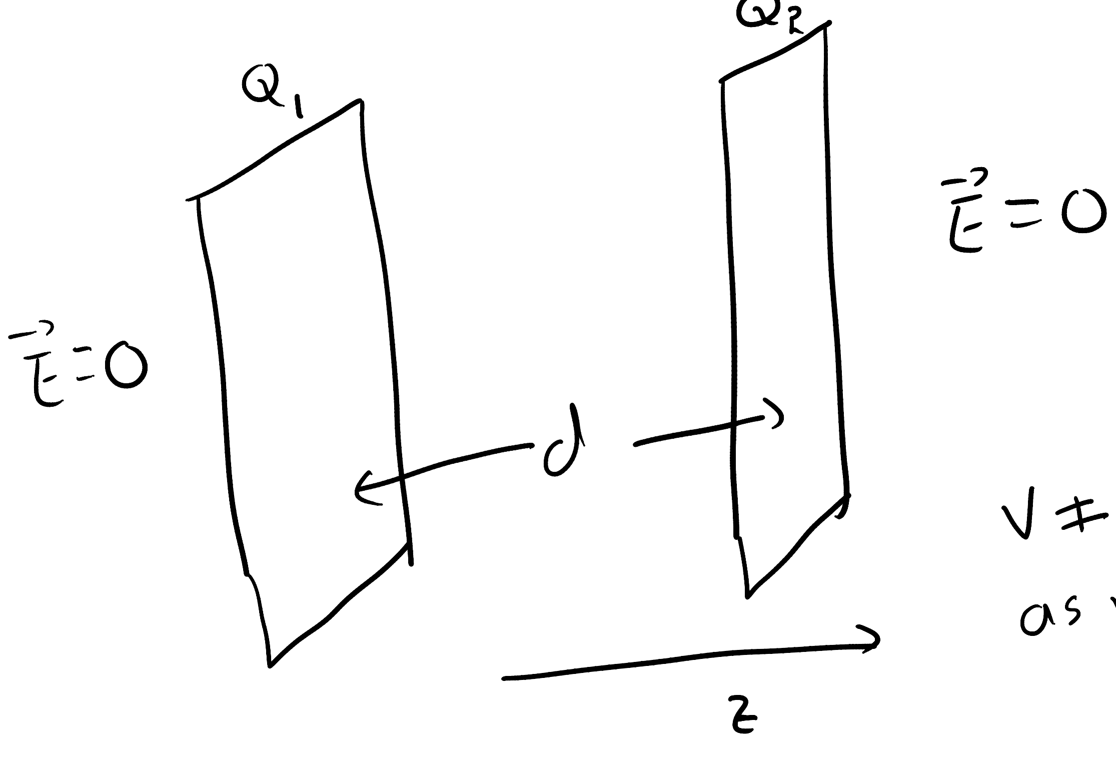
$$Q = C(V_1 - V_2)$$

$$C = \frac{C_{11}C_{22} - C_{12}^2}{C_{11} + C_{22} + 2C_{12}}$$

$$V_1 = B_{11} Q_1 + B_{12} Q_2$$
$$V_2 = B_{12} Q_1 + B_{22} Q_2$$

$$C = \frac{1}{B_{11} + B_{22} - 2B_{12}}$$

Parallel plate capacitor



$V \neq \infty$
as $r \to \infty$

Field due to Q_2 to the right

$$\vec{E} = \frac{\sigma_2}{2\varepsilon_0}\hat{z}$$

Field due to Q_1 to the right

$$\vec{E} = \frac{\sigma_1}{2\varepsilon_0}\hat{z}$$

($\vec{E} = 0$ implies $\sigma_1 = \sigma = -\sigma_2$)

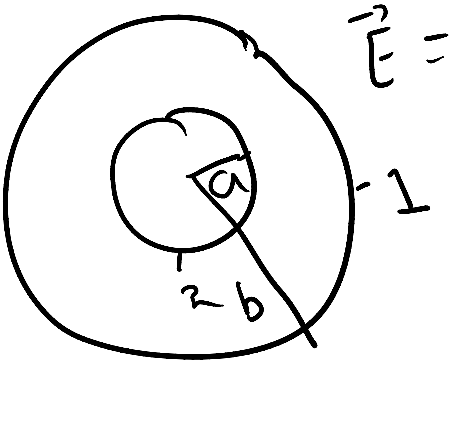
$\vec{E}$ in between $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{z}$

$$V_1 - V_2 = \int_1^2 \vec{E}\cdot d\vec{z} = \frac{\sigma}{\varepsilon_0}d = \frac{Q\,d}{A\varepsilon_0}$$

$$C = \frac{\varepsilon_0 A}{d}$$

Spherical Capacitor

$\vec{E} = 0$ outside

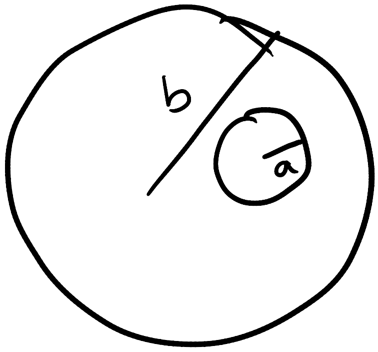


$$B_{11} = \frac{1}{4\pi\varepsilon_0 a} \; ; \; B_{22} = \frac{1}{4\pi\varepsilon_0 b}$$

$$B_{12} = \frac{1}{4\pi\varepsilon_0 b} = B_{21}$$

$$C = \frac{4\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}}$$

Off-centre



$$B_{22} = \frac{1}{4\pi\varepsilon_0 b}$$

$$B_{12} = \frac{1}{4\pi\varepsilon_0 b}$$

B_{11} complicated

$Q = CV$ (capacitor)

$$\frac{dQ}{dt} = C\frac{dV}{dt} \quad \left(\frac{dQ}{dt} = I\right)$$

$Q = CV$ derived from electrostatics,
so can we use it when $Q(t)$?
yes, if ω or $\frac{1}{\tau}$ are such that $\omega \ll \frac{c}{d}$ or
$\frac{1}{\tau} \ll \frac{c}{d}$, d - dimension