PH 206: Electromagnetic theory

Problem Set 3

Due date: Sat. Feb 25 2012

1. The Van der Waals interaction between molecules is electrostatic in nature. Assume that the molecules have polarizability α, i.e. upon the application of an electric field $\mathbf{E}$, the induced polarization of the molecule is $\mathbf{p} = \alpha\mathbf{E}$. The molecules have no permanent electric dipole moment. The interaction between two molecules is caused by one developing a dipole moment momentarily due to a fluctuation and inducing a dipole moment in the other a distance r away. The two dipole moments then interact with each other. The average dipole moment over all fluctuations is zero but the mean square moment is η. Further, the molecules are completely isotropic in space as are the fluctuations. What is the Vand der Waals interaction potential as a function of the distance r? Is the resultant force attractive or repulsive?

2. In this problem you will calculate the field produced due to a dielectric sphere of radius R in two different situations.

   (a) First consider a dielectric sphere of permittivity ϵ which is placed in a uniform electric field $\mathbf{E}_0$, i.e. had the sphere not been present the field in all of space would have been $\mathbf{E}_0$. Calculate the field $\mathbf{E}$, the displacement vector $\mathbf{D}$ and polarization density $\mathbf{P}$ everywhere. What is the total induced dipole moment in the sphere and what are the volume and surface bound charge densities?

   (b) Now, assume that there is no applied electric field but the sphere has a uniform polarization density "frozen in" such that the total dipole moment is the same as in the previous part. Calculate $\mathbf{E}$, $\mathbf{D}$ and the volume and surface bound charge densities.

3. In this problem you will derive expressions for the work done in setting up dipole moments in dielectrics and magnetic materials.

   (a) First consider a parallel place capacitor connected to a battery so that there is an electric field $\mathbf{E_0}$ between the plates. A dielectric slab of volume V is introduced between the two plates which causes it to develop a uniform polarization density $\mathbf{P}$. Show that the work done by the battery to change the polarization by an amount $d\mathbf{P}$ is $dW = \mathbf{E_0}.d\mathbf{P}V$. (*Hint: If the polarization of the slab changes, it will try change the potential difference $\Delta\phi$ between the plates. However, $\Delta\phi$ cannot change since the plates are connected to a battery of a fixed voltage. The only way $\Delta\phi$ can remain constant is if the charge on the plates changes to counter the effect of the changing polarization. Thus the work done by the battery is the work done to supply the extra charge to the plates.*)

   (b) Now consider the magnetostatic analogue of the above situation. An infinitely long solenoid is connected to a current source that supplies a constant current. This results in a magnetizing field $\mathbf{H}_0$ inside the solenoid. A magnetic material of volume V in the shape of a solid cylinder is placed inside the solenoid parallel to its axis so that it develops a uniform magnetization density $\mathbf{M}$. Show that the work done by the current source to change the magnetization by an amount $d\mathbf{M}$ is $dW = -\mu_0\mathbf{H}_0.d\mathbf{M}V$.

4. Prove that

$$\int_\Omega (\mathbf{r}.\mathbf{r}')\,\mathbf{J}(\mathbf{r}')d\mathbf{r}' = -\int_\Omega \mathbf{r}'\,[\mathbf{r}.\mathbf{J}(\mathbf{r}')]\,d\mathbf{r}',$$

   for a current density $\mathbf{J}(\mathbf{r}')$ localized to the volume Ω which produces a static magnetic field. i.e. $\boldsymbol{\nabla}\times\mathbf{B}(\mathbf{r}') = \mu_0\mathbf{J}(\mathbf{r}')$ in Ω. Recall that this is the relation we had used to derive the magnetic dipole term in the multipole expansion for the magnetic vector potential.

5. This problem is the same as problem 2 but for magnetostatics instead of electrostatics.

   (a) A sphere of radius R made of a magnetic material of permeability μ is placed in a uniform magnetic field $\mathbf{B}_0$. Calculate $\mathbf{B}$, $\mathbf{H}$ and $\mathbf{M}$ everywhere in space. What is the total induced dipole moment in the sphere and the bound volume and surface current densities?

   (b) Now, assume there is no external field but the sphere has a frozen in magnetization $\mathbf{M}$, which gives the same dipole moment as in the previous part. Calculate $\mathbf{B}$, $\mathbf{H}$ and $\mathbf{M}$ everywhere in space and the bound currents.