PH 206: Electromagnetic theory

Problem Set 1

Due date: Mon. Jan 30 2012

1. A constant charge density ρ fills *all of space*.

   (a) What do you expect the value of the electric field $\mathbf{E}(\mathbf{r})$ at a point $\mathbf{r}$ to be from considerations of "symmetry"?

   (b) Does the value of the electric field you obtained in the previous part obey Gauss's law $\boldsymbol{\nabla}.\mathbf{E}(\mathbf{r}) = \rho/\epsilon_0$? What do you think is the $\mathbf{E}(\mathbf{r})$ that satisfies Gauss's law?

   (c) What value of $\mathbf{E}(\mathbf{r})$ will you obtain if you calculate it from the integral

   $$\mathbf{E}(\mathbf{r}) = \frac{\rho}{4\pi\epsilon_0}\int \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}d\mathbf{r}'?$$

   Does this agree with what you get in parts (a) and (b)? If not, how do you reconcile the different values with one another?

2. In class it was shown that for a set of conductors, the following relation holds between their potentials and charges

   $$Q_i = \sum_j C_{ij}\phi_j,$$

   where ϕ_i is the potential of the surface of the i^{th} conductor and Q_i is the charge on it. This equation can be inverted to obtain

   $$\phi_i = \sum_j B_{ij}Q_j.$$

   Show that $C_{ij} = C_{ji}$ and $B_{ij} = B_{ji}$.

3. A balloon made of a conducting material of surface tension T is initially completely deflated (has zero volume). The balloon is then slowly charged until the charge on it is q. Assume that the balloon is always spherical during the charging process.

   (a) What is the radius of the balloon after it has been fully charged?

   (b) What is the total electrostatic energy ϵ calculated according to the relations

   $$\epsilon = \frac{\epsilon_0}{2}\int_{allspace}\mathbf{E}^2 d\mathbf{r}$$

   and

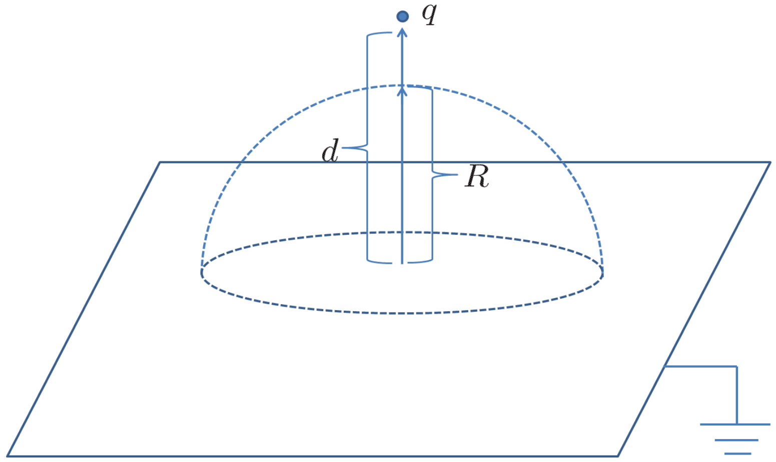
   $$\epsilon = \int \rho V d\mathbf{r},$$

   where the symbols have their usual meanings? Does the energy obtained from the two expressions agree with each other?

   (c) What is the total work done W to charge the balloon?

   (d) Does W obtained in part (c) agree with the values of ϵ obtained in part (b)? If not, how will you resolve the discrepancy?

4. An infinite grounded conducting plane has a hemispherical bump of radius R. A point charge q is placed at a distance d above the centre of the hemisphere as shown in the figure (the line joining the charge and the centre of the hemisphere is perpendicular to the conducting plane).



(a) Calculate the electric field $\mathbf{E}(\mathbf{r})$ everywhere.

(b) What is the total charge on the conducting plane (with the bump)?

5. Show that if $V(\mathbf{r})$ obeys Laplace's equation in a volume Ω,

$$V(0) = \frac{1}{4\pi R^2} \iint V(\mathbf{r}) dS,$$

where the integral is over the surface of a sphere of radius R contained in Ω and $V(0)$ is the value of $V(\mathbf{r})$ at its centre. *Hint: Write* $V(0) = \int V(\mathbf{r})\delta(\mathbf{r})d\mathbf{r} = -\frac{1}{4\pi}\int V(\mathbf{r})\nabla^2\left(1/r\right)d\mathbf{r}$ *where the integration is over the volume of the sphere and use results from vector calculus.*

6. The northern and southern hemispheres of a spherical shell have uniform surface charge density σ and -σ respectively. Let $V(r,\theta,\phi)$ in spherical polar coordinates with the origin at the origin of the sphere.

(a) What is the leading order term in the potential as $r \to \infty$? Calculate the first three sub-leading order terms.

(b) What is the leading order term in the potential as $r \to 0$? Calculate the first three sub-leading order terms.