Angular momentum transport in accretion

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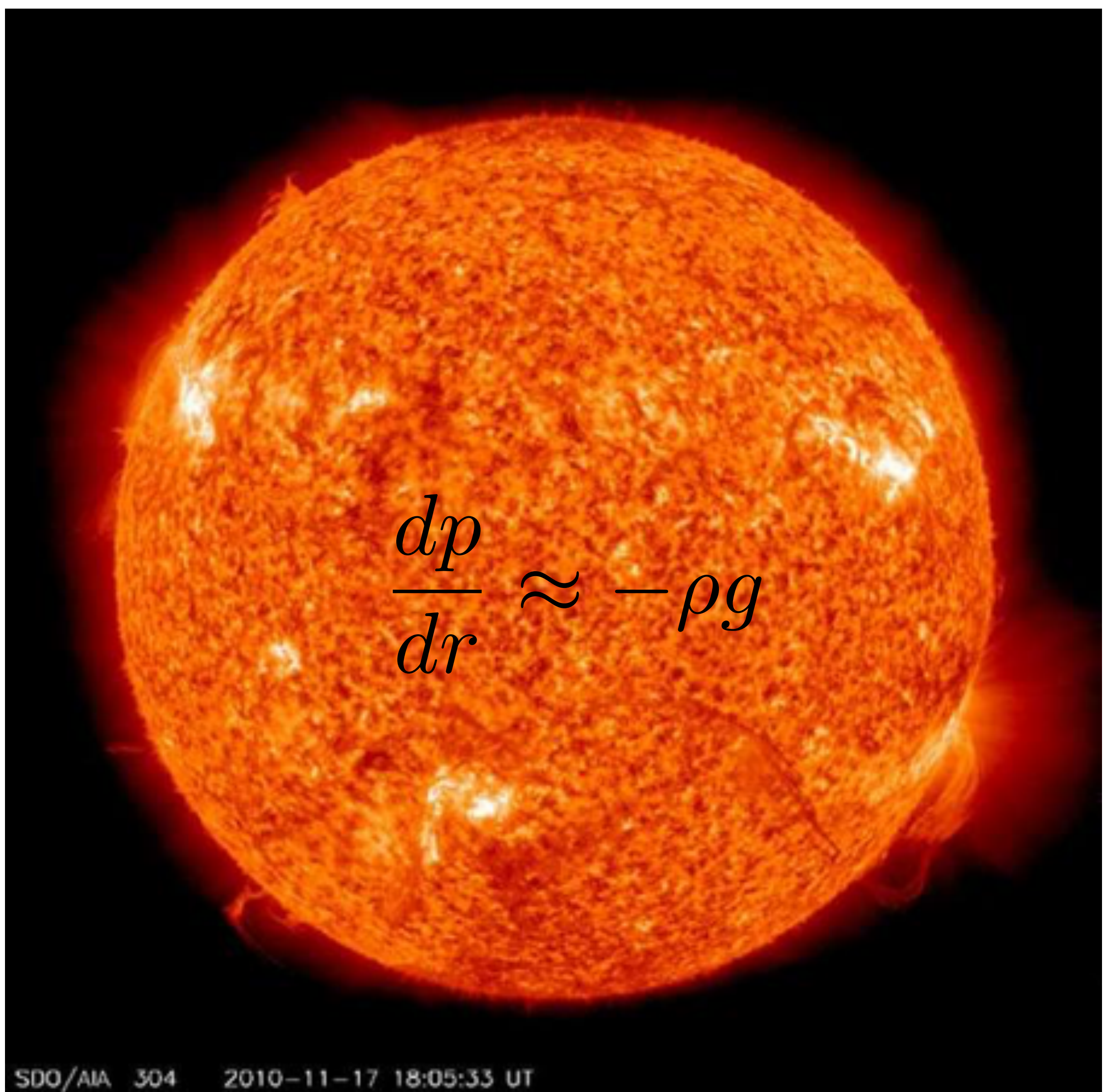
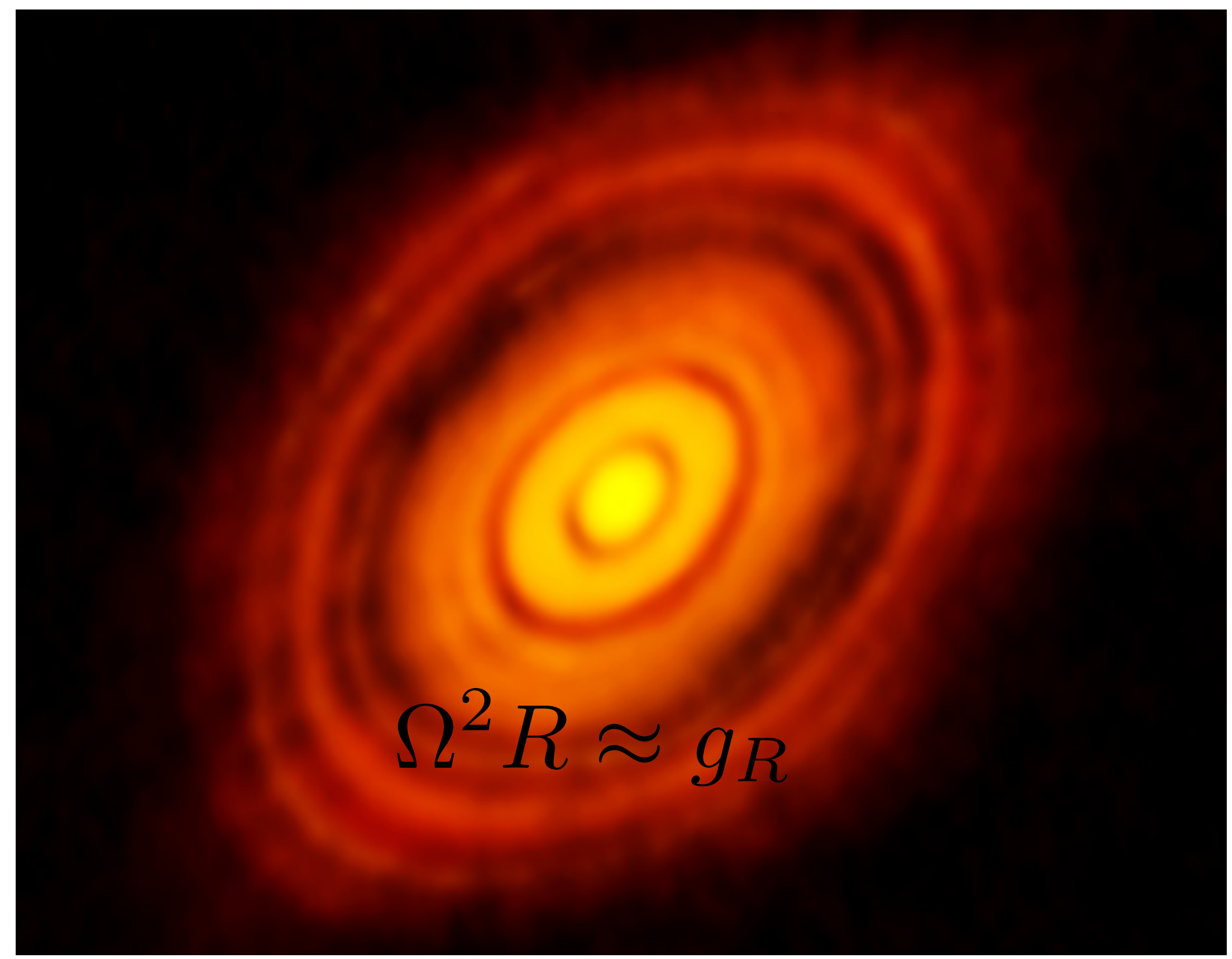
Accretion

Two major classes of objects in astrophysics:

pressure supported, e.g. stars

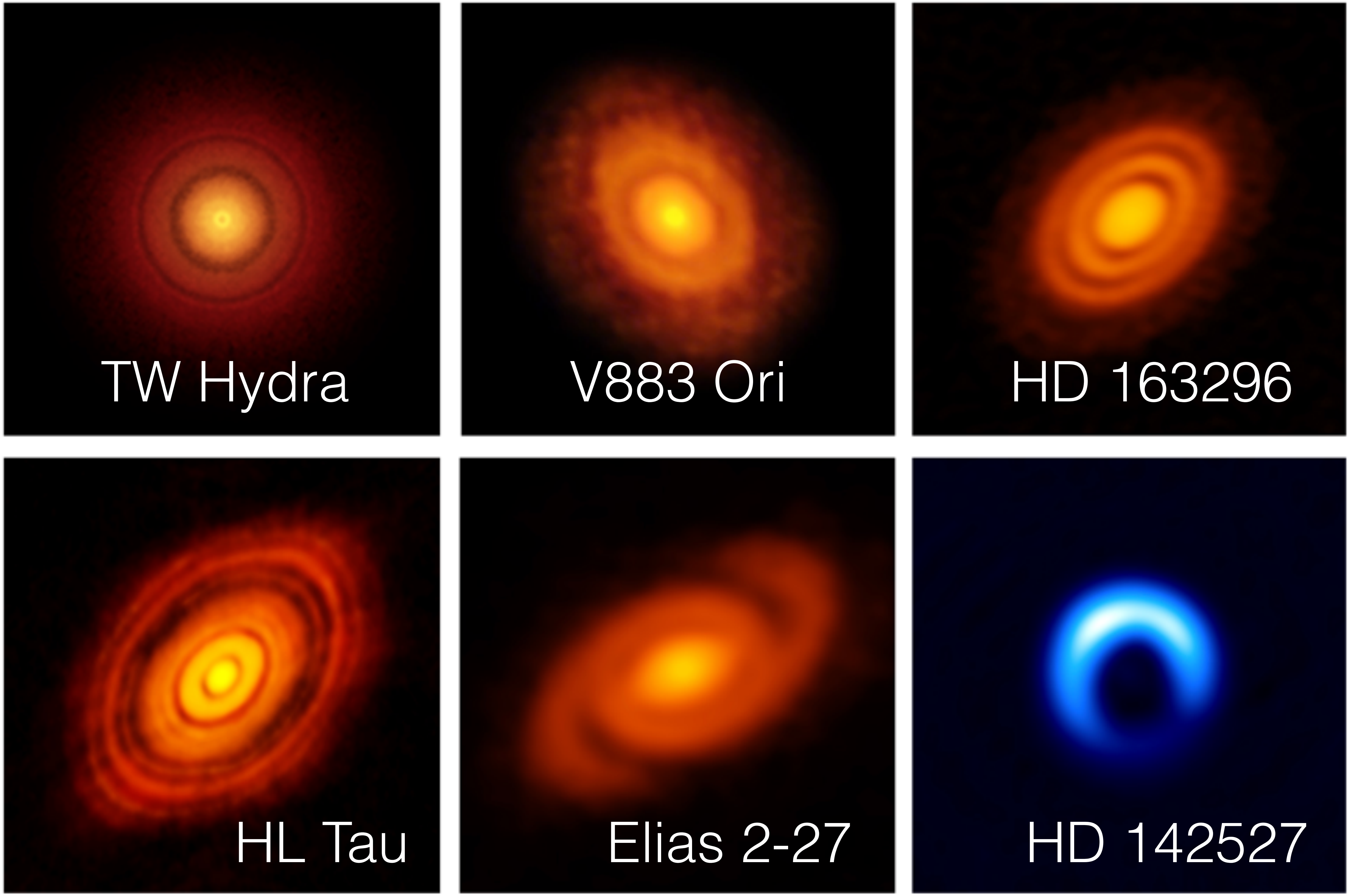


rotation supported, e.g. disks



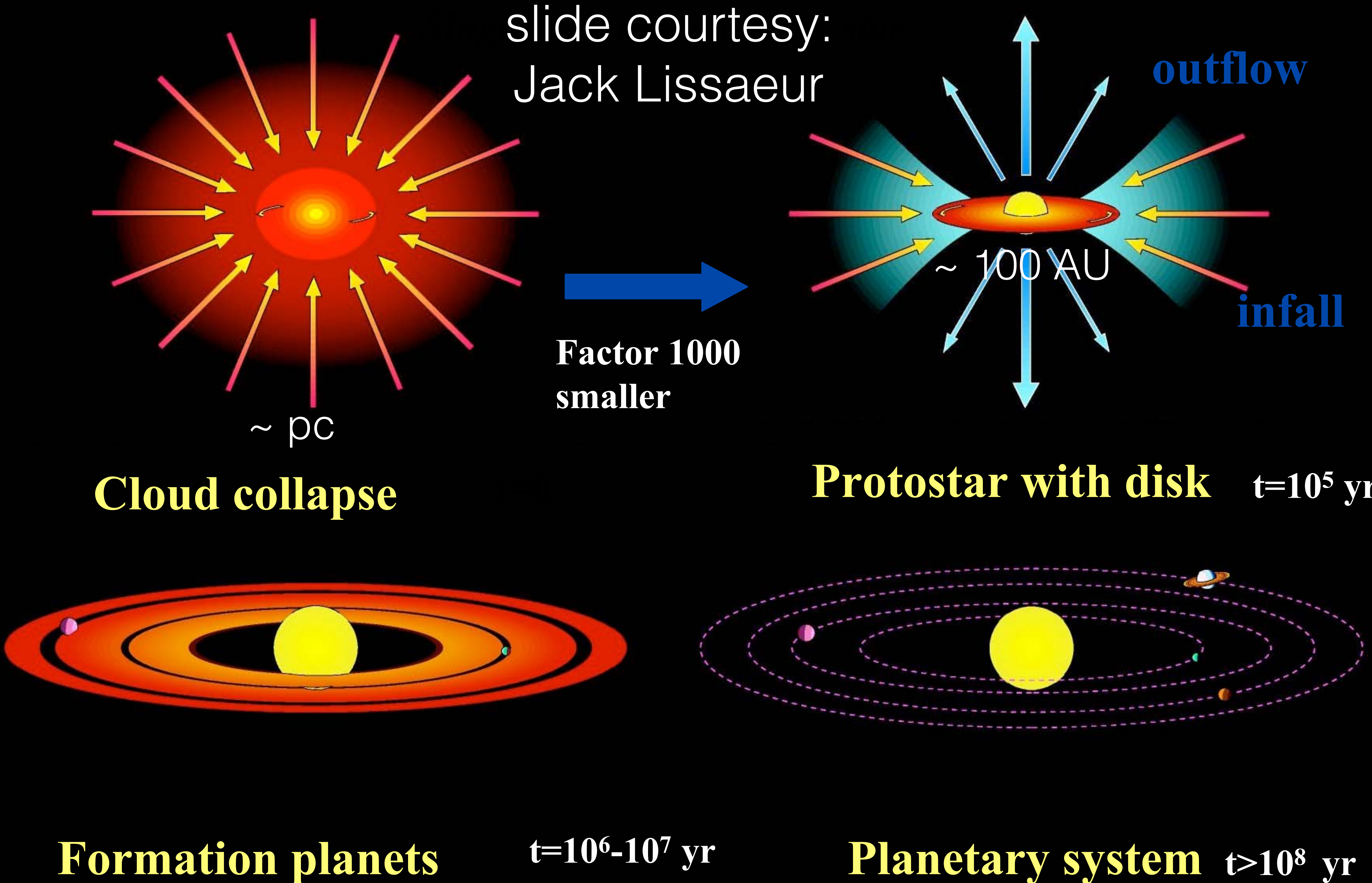
formed because collapsing matter lost energy but not angular momentum

ALMA disks

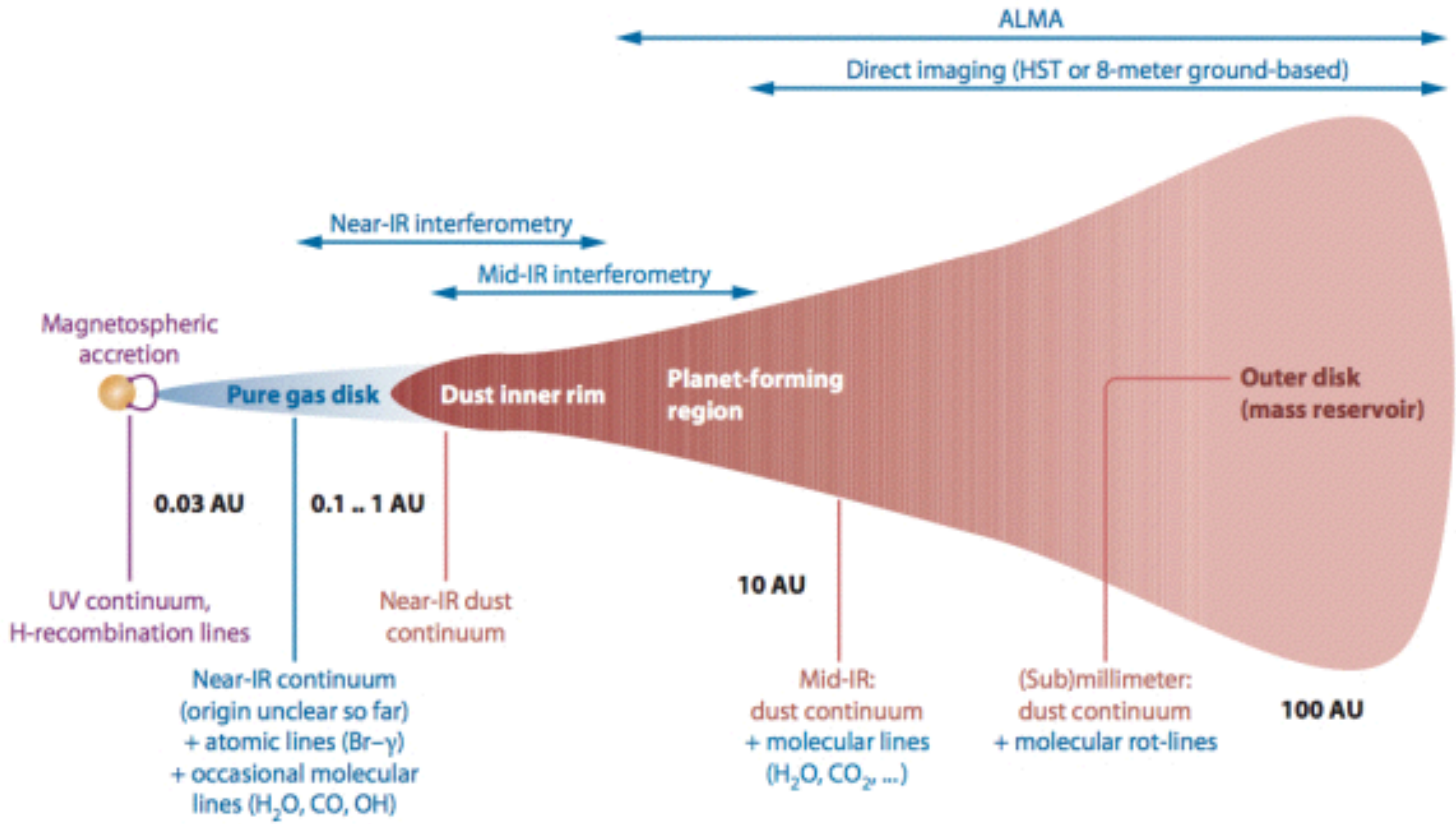


sub-mm emission from dust in protoplanetary disks ~ 100 AU

Scenario for star- and planet formation



Protoplanetary disk

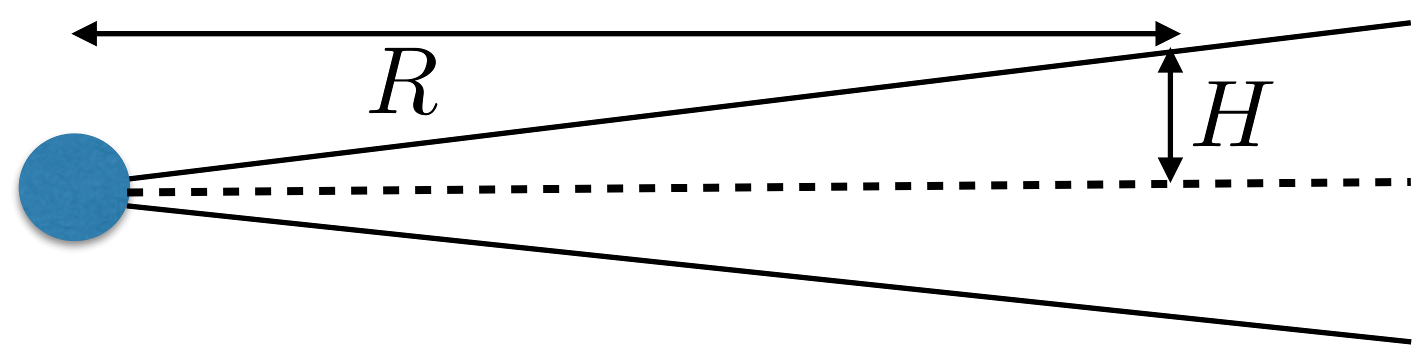


Equilibrium structure

gravity dominated by central object

$$\frac{GM}{R^2} \approx \Omega^2 R$$ radial force balance => Keplerian flow

$$\Omega_K = \sqrt{\frac{GM}{R^3}} \propto R^{-3/2}$$



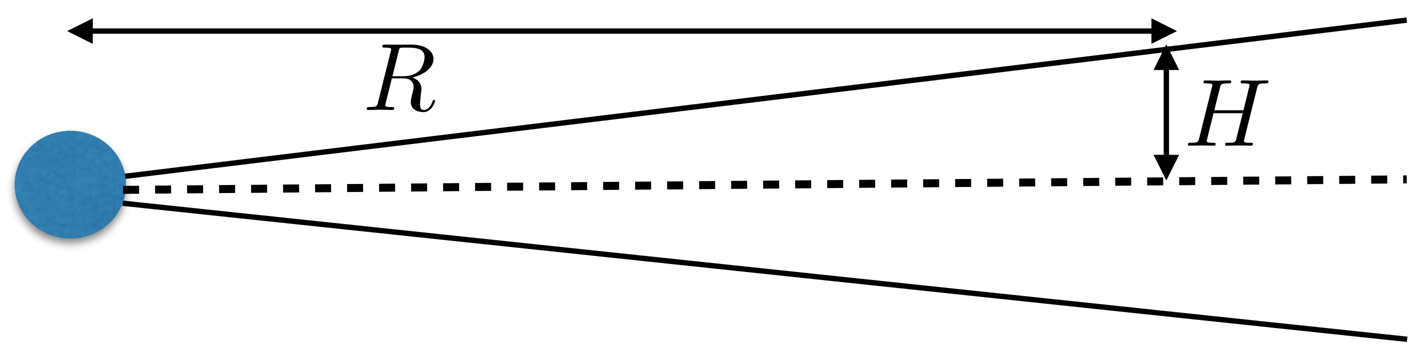
$$\frac{1}{\rho}\frac{dp}{dz} \approx -\frac{GMz}{R^3} = -\Omega_K^2 z$$ vertical HSE => $H \approx c_s/\Omega_K$

$$H/R \approx c_s/v_K \ll 1$$

dynamical equilibrium & **slow** radial accretion of matter

radial & vertical dynamics decoupled => 1-D solutions vs. R of z-integrated quantities

Energy considerations

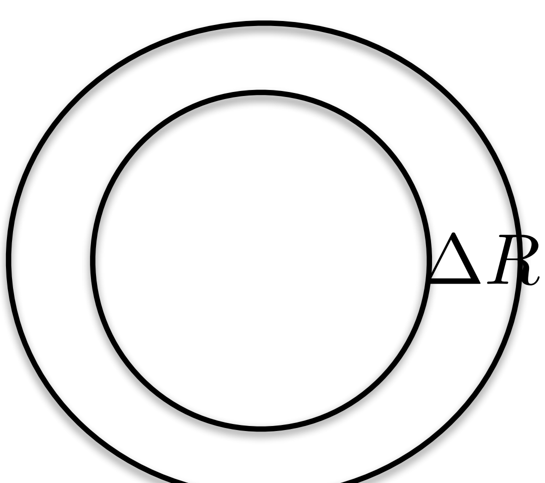


total energy per mass for an orbiting particle= $-\dfrac{GM}{2R}$

KE in rotation= $\dfrac{GM}{2R}$ gravitational PE= $-\dfrac{GM}{R}$

$\dfrac{GM}{2R}$ energy per accreted mass available as heat & light

accretion power generated from R to R+ΔR:



$$GM\dot{M}\Delta\left(\frac{-1}{2R}\right) = \frac{GM\dot{M}}{2R^2}\Delta R$$

total power released: $\dfrac{GM\dot{M}}{2R_{\rm in}}$

Accretion power

$$\frac{GM\dot{M}}{2R_{\rm in}} = \eta \dot{M} c^2$$

accretion efficiency

$$\eta = \frac{GM}{2R_{\rm in}c^2} = \frac{v_{\rm esc}^2}{4c^2} = \frac{R_{\rm Sch}}{4R_{\rm in}}$$

higher efficiency for compact accretor

$4p \rightarrow {}^4He + 2e^+ + 2\nu_e$

+27 MeV

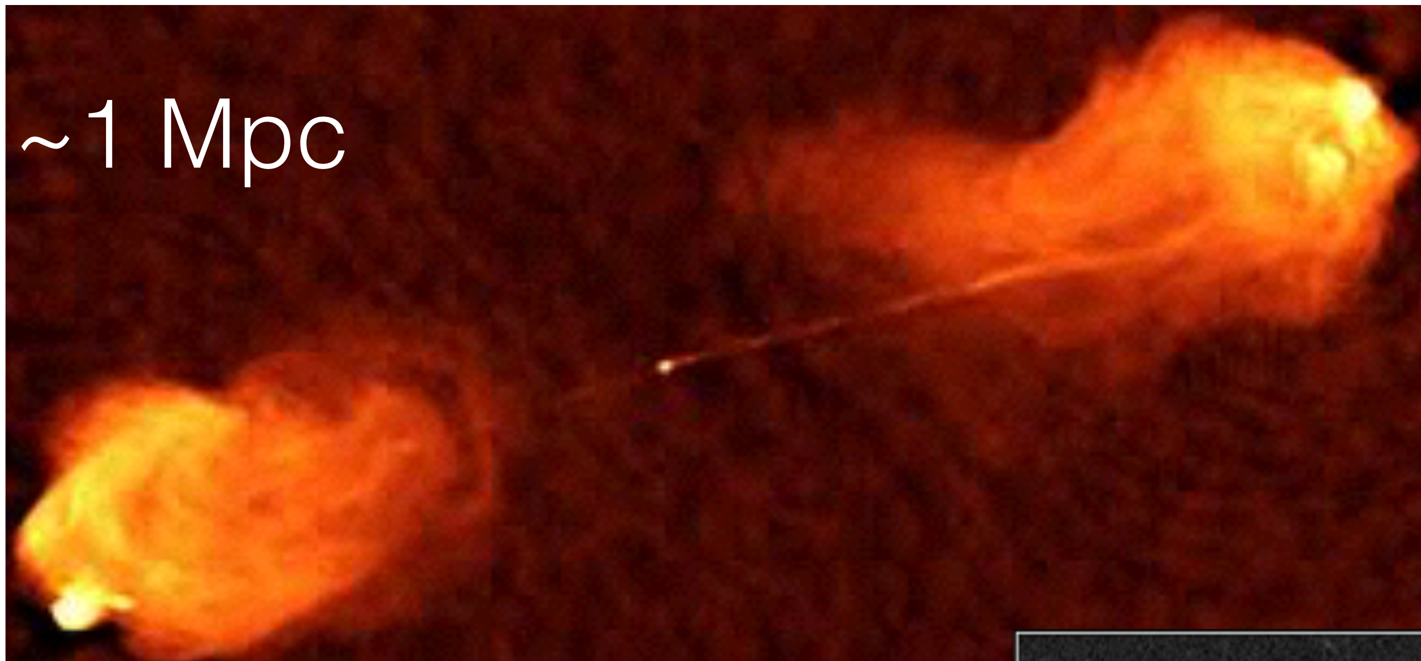
$$\eta_{\rm nuc} \approx 27\ {\rm MeV}/4\ {\rm GeV}$$

= 0.007

no wonder most powerful sources due to accretion!

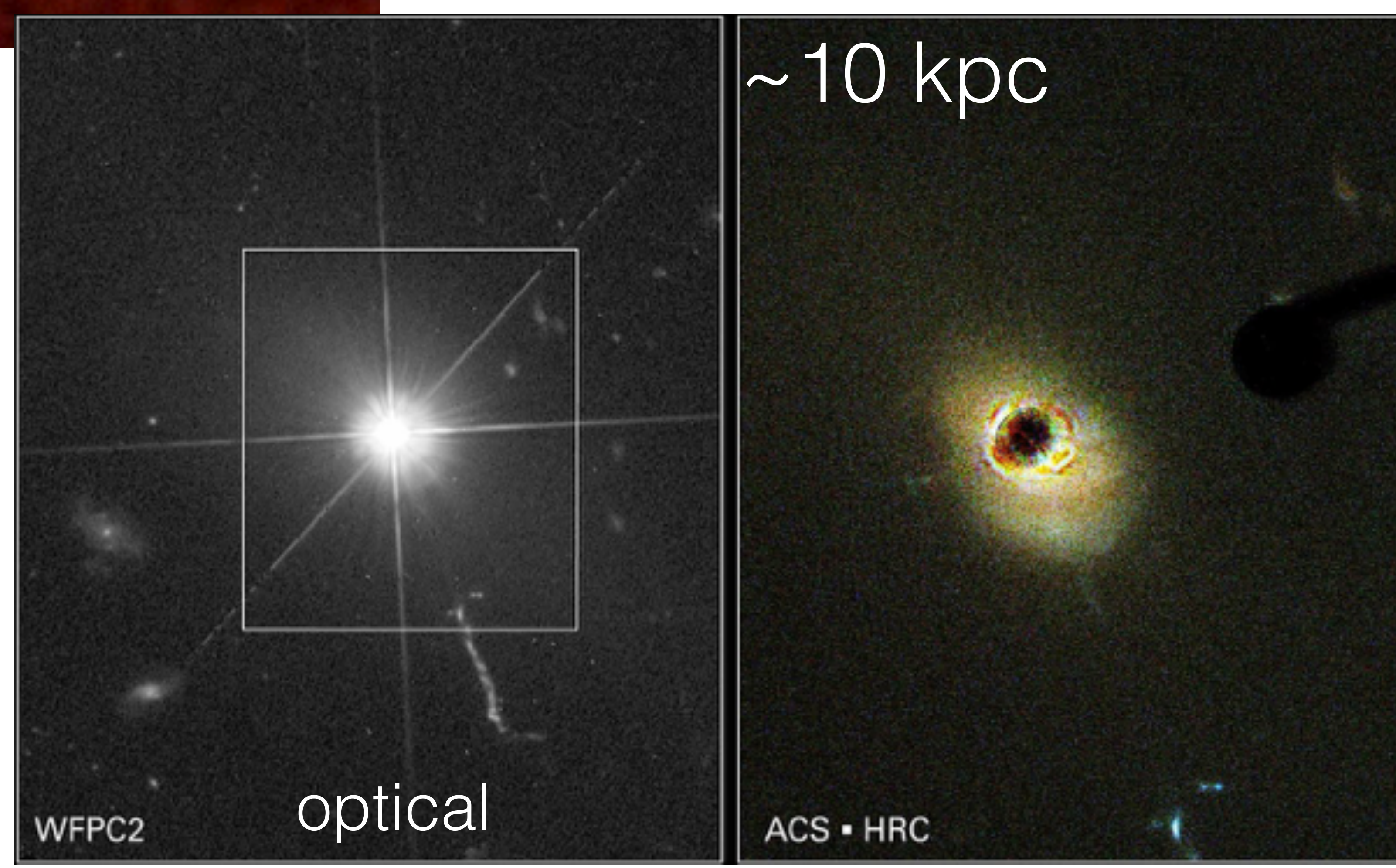
	$R_{\rm in}({\rm km})$	$M(M_\odot)$	$R_{\rm in}/R_{\rm Sch}$	η
protostar	10^6	1	3×10^5	10^{-6}
WD	10^4	1	3000	10^{-4}
NS	10	1	3	0.1
BH	$\frac{9M}{M_\odot}$	10, 10^{6-9}	3	0.1

Manifestation



quasar 3C-273 in a galaxy
radiative power

radio galaxy, Cyg A
mechanical power
synchrotron radiation
from relativistic e^-s



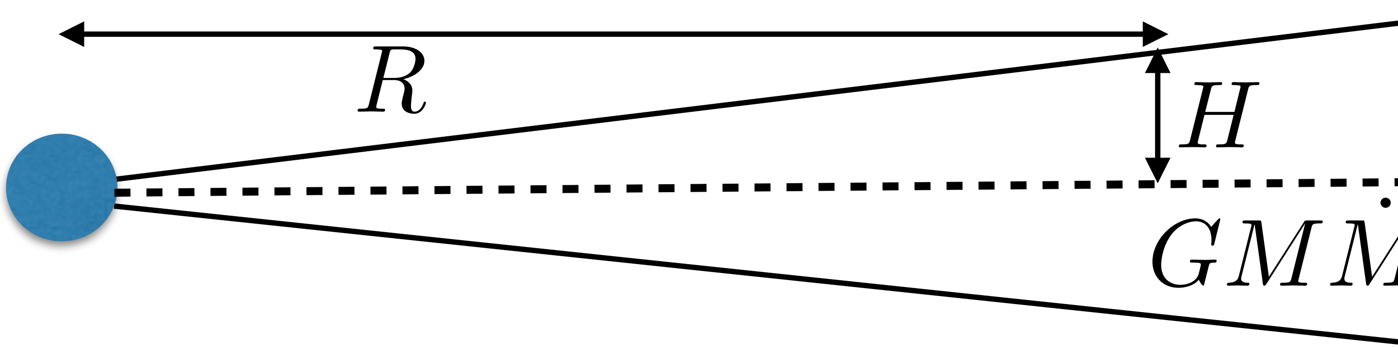
Flow Temperature

Two estimates:

virial applicable for non-radiative disks

$$k_B T_{\text{virial}} \equiv GMm_p/3R \sim 0.1 m_p c^2 \sim 10^{12}\text{K}$$

geometrically thin, optically thick disk, a local BB disk



$$GM\dot{M}\Delta\left(\frac{-1}{2R}\right) \sim 2\pi R \Delta R \sigma T^4$$

T~1 keV for maximally accreting 10 solar mass BH, emits X-rays

Angular momentum transport

Key Q: how does matter lose angular momentum and fall in?
"molecular viscosity" negligible, need turbulent transport, **but how**?

Linear response: $w^2 = \frac{k_z^2}{k^2}\kappa^2, \ \kappa^2 = \frac{2\Omega}{R}\frac{dl}{dR}$

$l = \sqrt{GMR}$

stable inertial waves
or epicycles oscillations

a perturbed fluid element preserves angular momentum
& **hydro Keplerian disks are Rayleigh stable**
(to local axisymmetric modes)

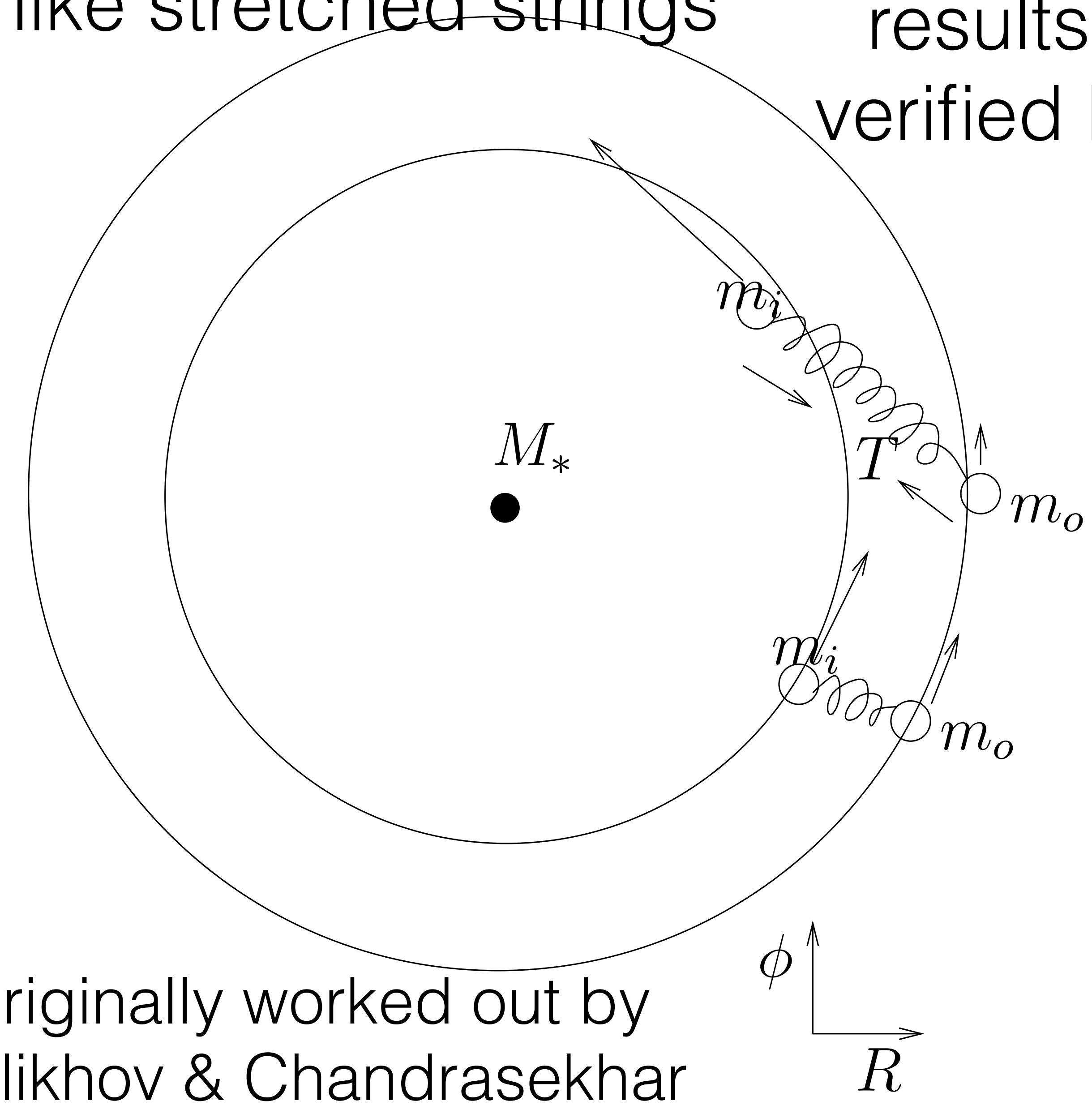
Hydro Keplerian flow does not naturally break into turbulence

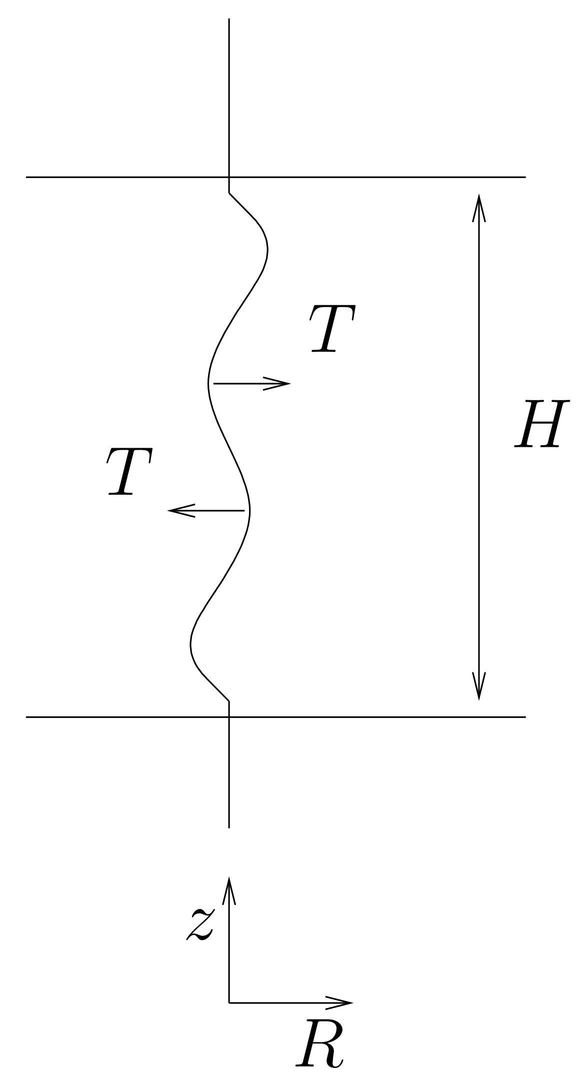
Linear MHD instability

[Balbus & Hawley 1991]

local axisymmetric MHD => MRI
magnetorotational instability
works for ionized flows
results in **MHD turbulence & transport**
verified by local & global MHD sims [DNS]

magnetic fields behave
like stretched strings





originally worked out by
Velikhov & Chandrasekhar

Linear MHD instability

[Balbus & Hawley 1991]

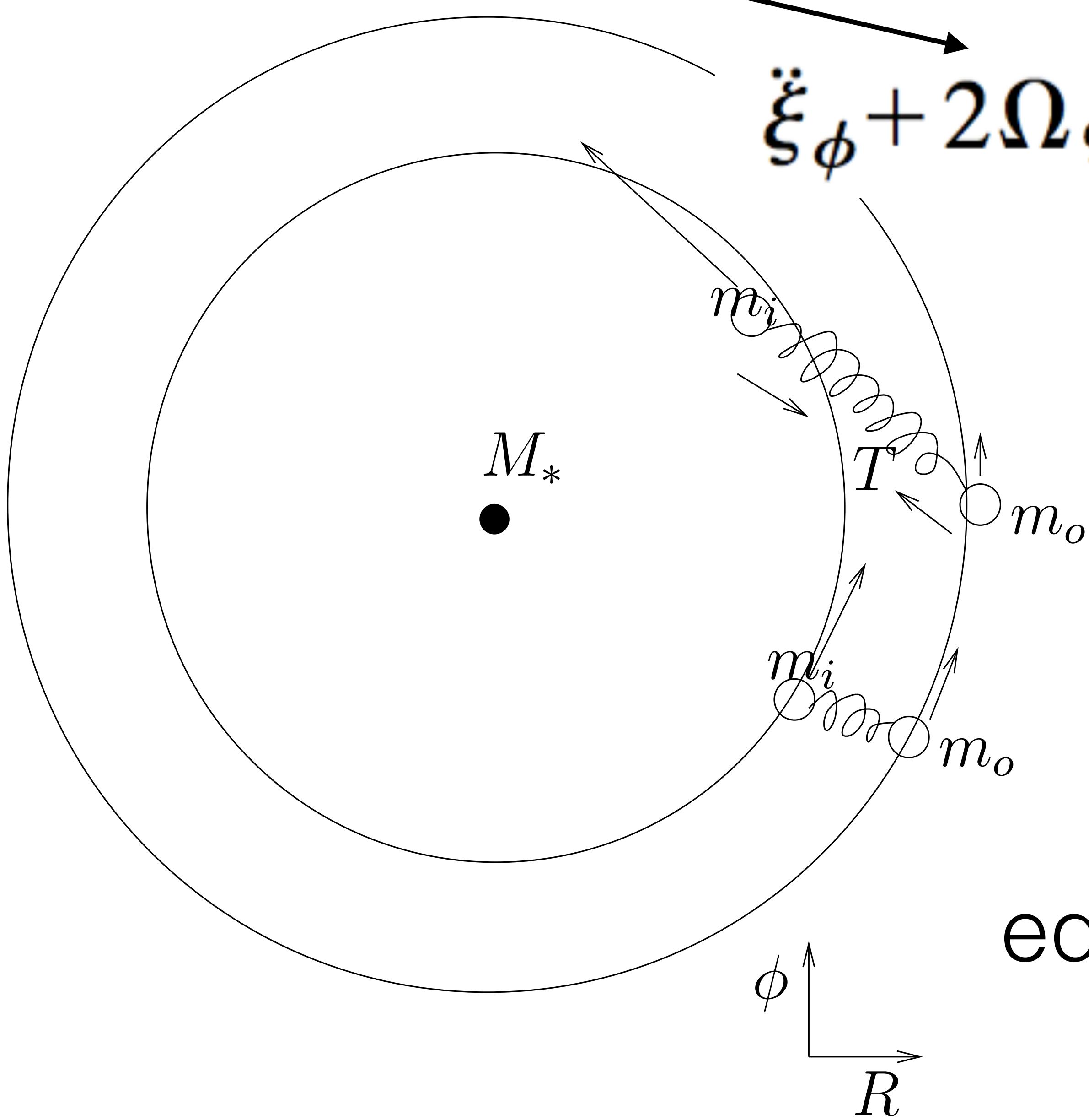
tidal force

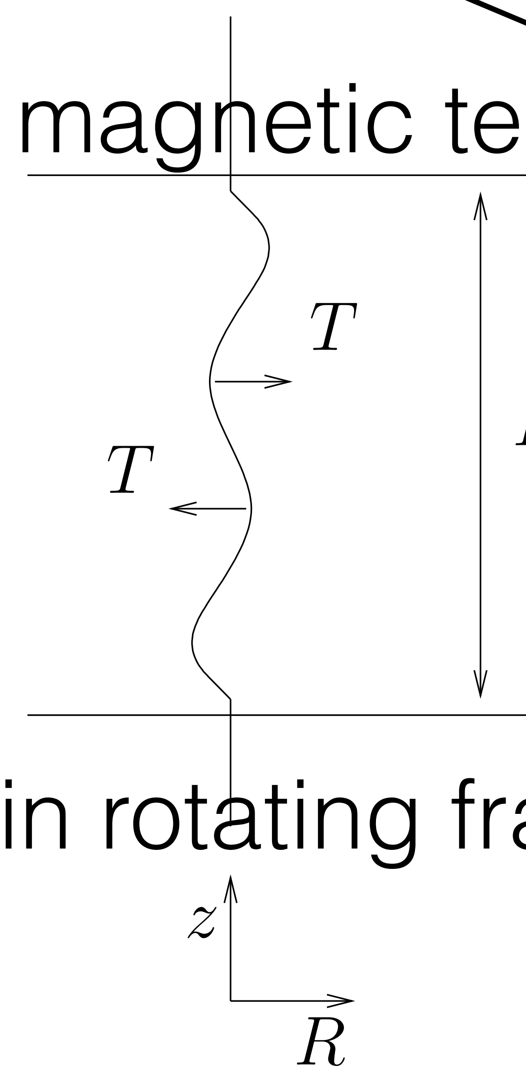
$$\ddot{\xi}_R - 2\Omega\dot{\xi}_\phi = -\left(\frac{d\Omega^2}{d\ \ln\ R} + (\mathbf{k}\cdot\mathbf{u_A})^2\right)\xi_R\,,$$

coriolis force

$$\ddot{\xi}_\phi + 2\Omega\dot{\xi}_R = -(\mathbf{k}\cdot\mathbf{u_A})^2\xi_\phi\,.$$

magnetic tension force

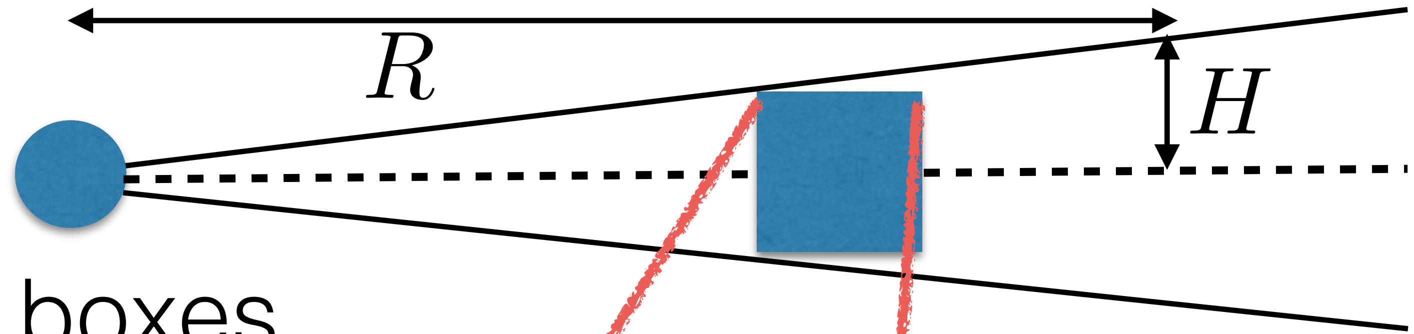


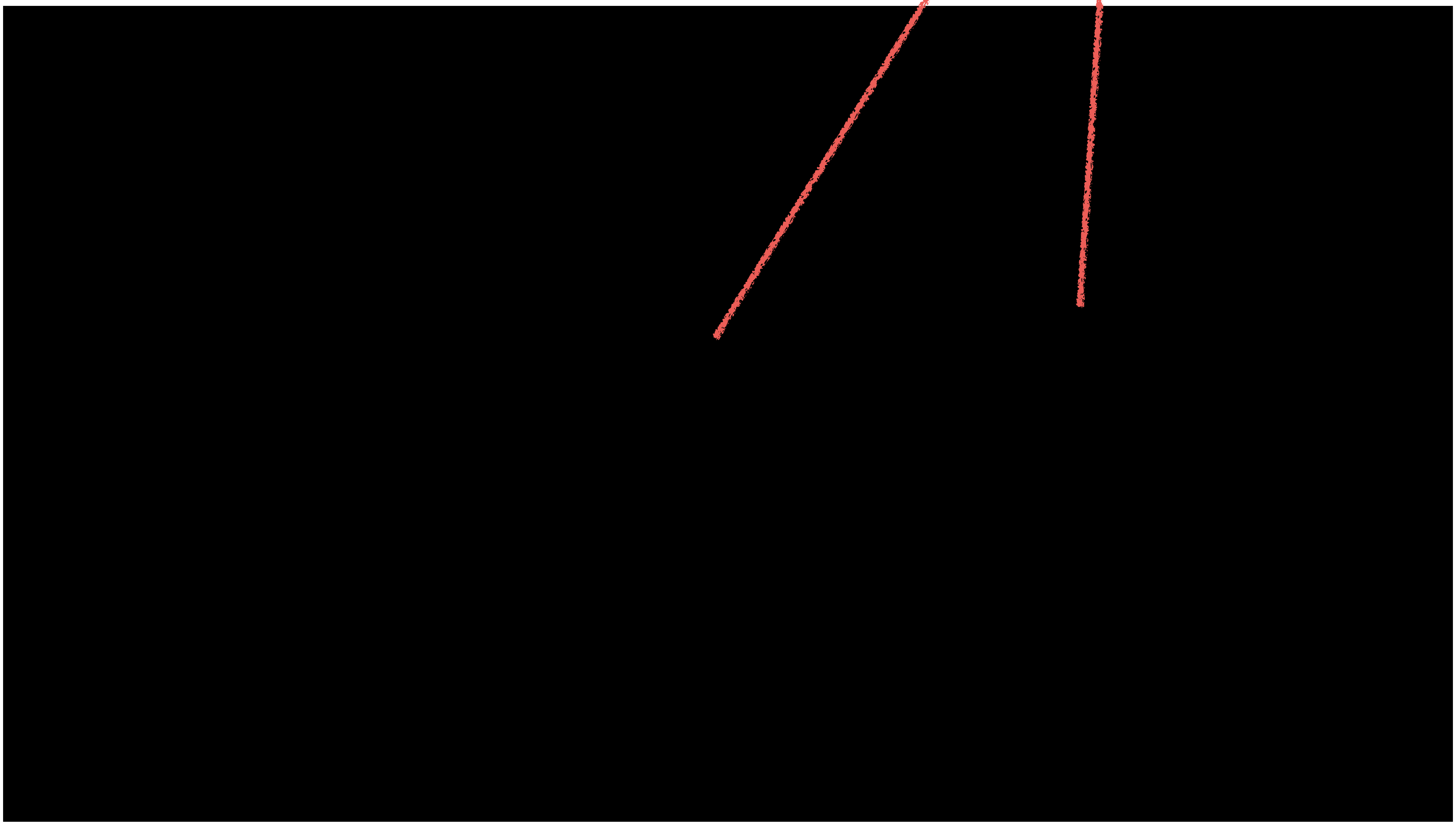


equations in rotating frame

MHD sims

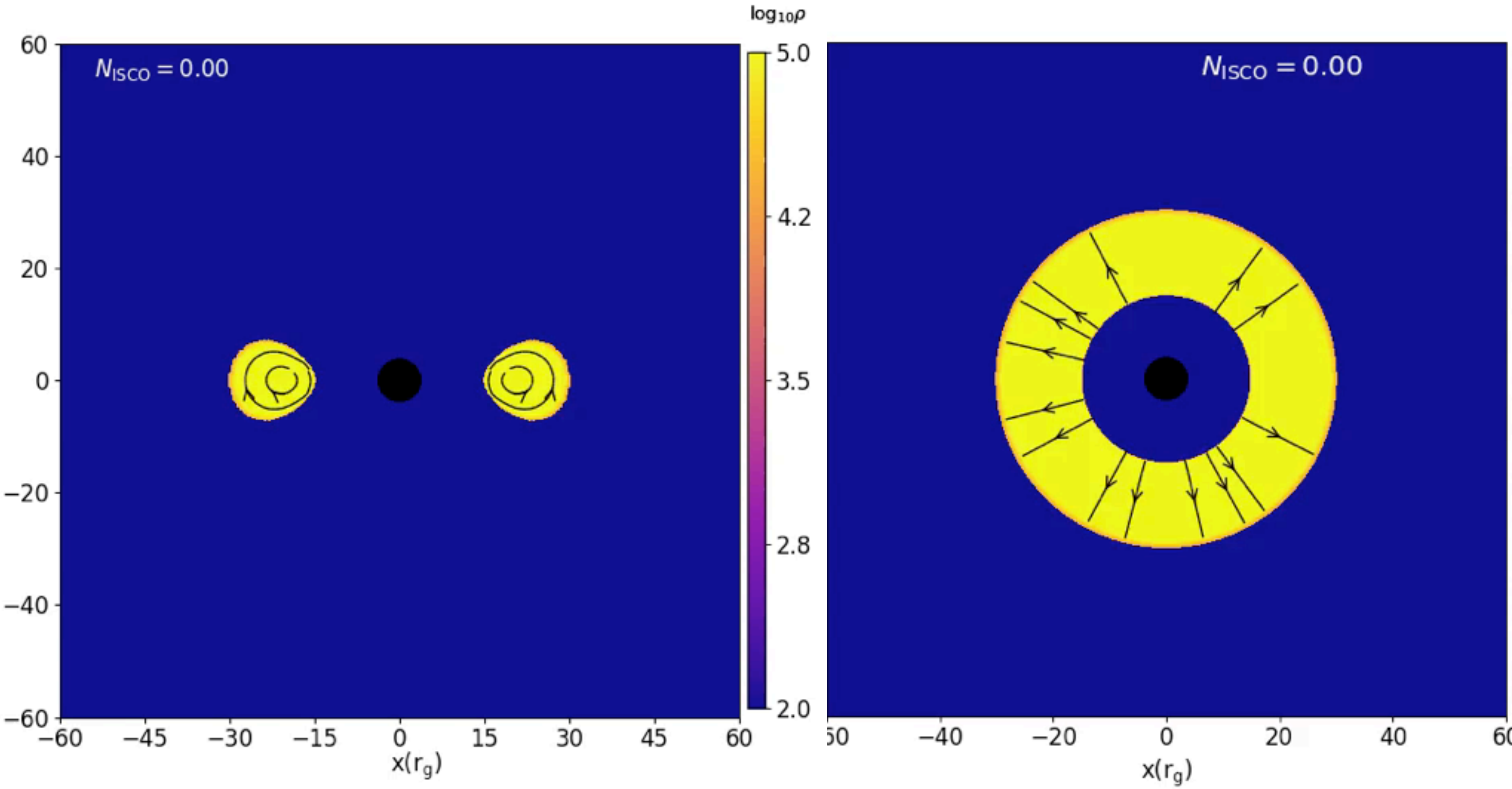
local shearing boxes



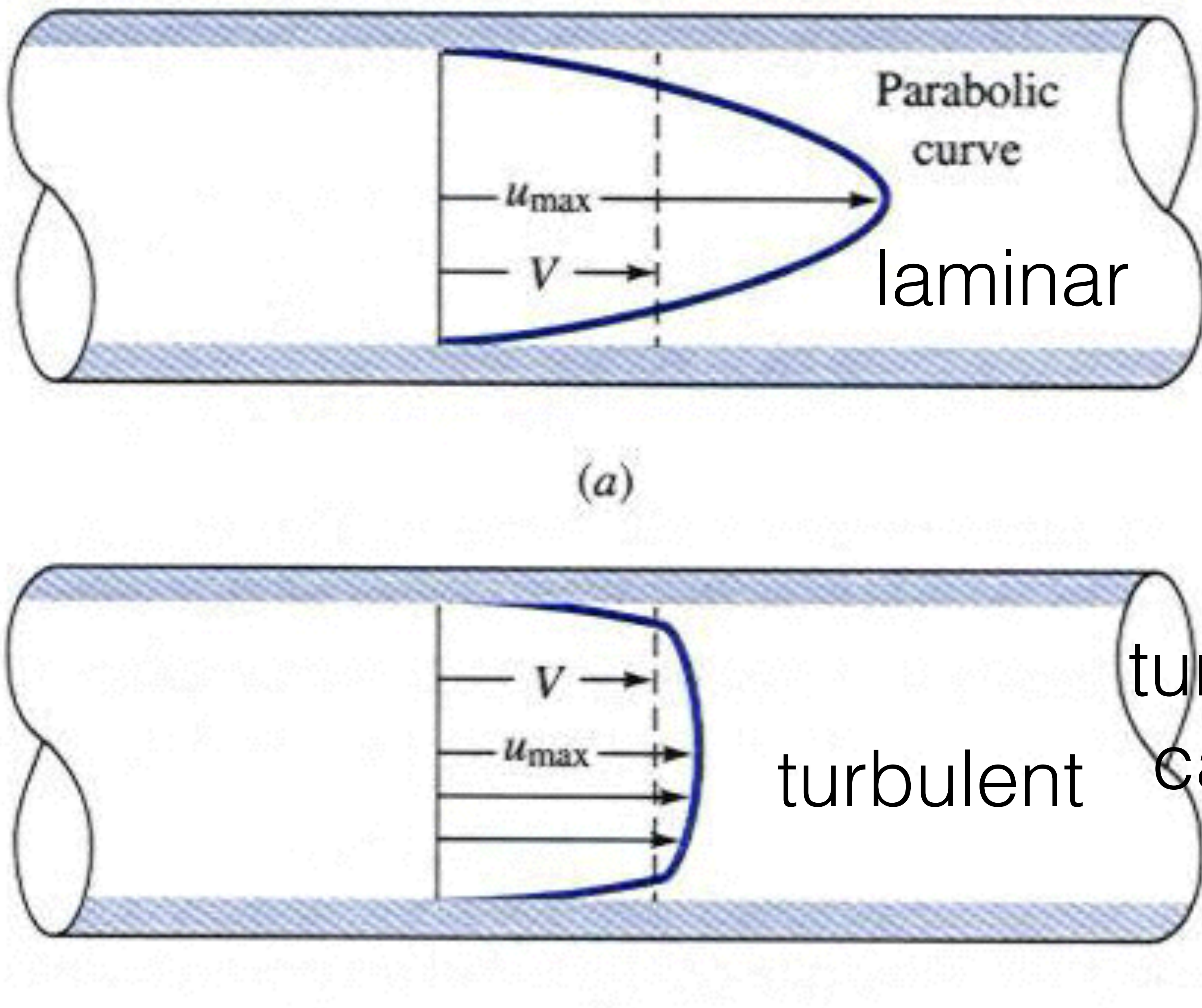


Global simulations

[movies by Prasun Dhang]



Turbulent transport



Pipe flow:
stream-wise turbulent momentum transport => flattening of velocity profile

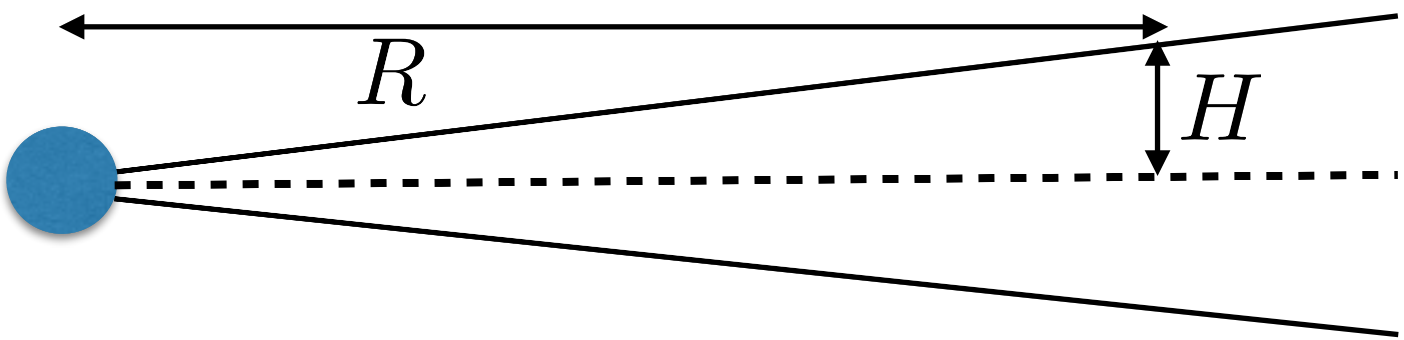
turbulent correlations $\langle u_R u_Z \rangle$ cause momentum transport from center to wall

Turbulent Reynolds stress modifies mean flow

Angular momentum eq.

inviscid, ϕ-averaged angular momentum equation

$$\frac{\partial}{\partial t}(\rho\langle u_\phi\rangle R) + \frac{1}{R}\frac{\partial}{\partial R}\left[R^2\left(\langle \rho u_\phi u_R - \frac{B_\phi B_R}{4\pi}\rangle\right)\right] + \frac{\partial}{\partial z}\left(R\langle \rho u_\phi u_z - \frac{B_\phi B_z}{4\pi}\rangle\right) = 0$$



$$\Sigma \equiv \int_{-H/2}^{H/2} \rho dz$$

$$\dot{M} \equiv 2\pi R \Sigma \langle u_R\rangle$$

in SS, integrating over z, angular momentum flow rate is const.

$$R^2\left[\Sigma\langle u_R\rangle\langle u_\phi\rangle + \langle \Sigma u_{R1} u_{\phi 1} - H\frac{B_R B_\phi}{4\pi}\rangle\right]$$

Reynolds & Maxwell stress

$$W_{R\phi} = \frac{\dot{M} l}{2\pi R^2 H}\left(1 - \frac{l_\star}{l}\right)$$

non-zero turbulent stress needed for mass accretion!

Angular momentum

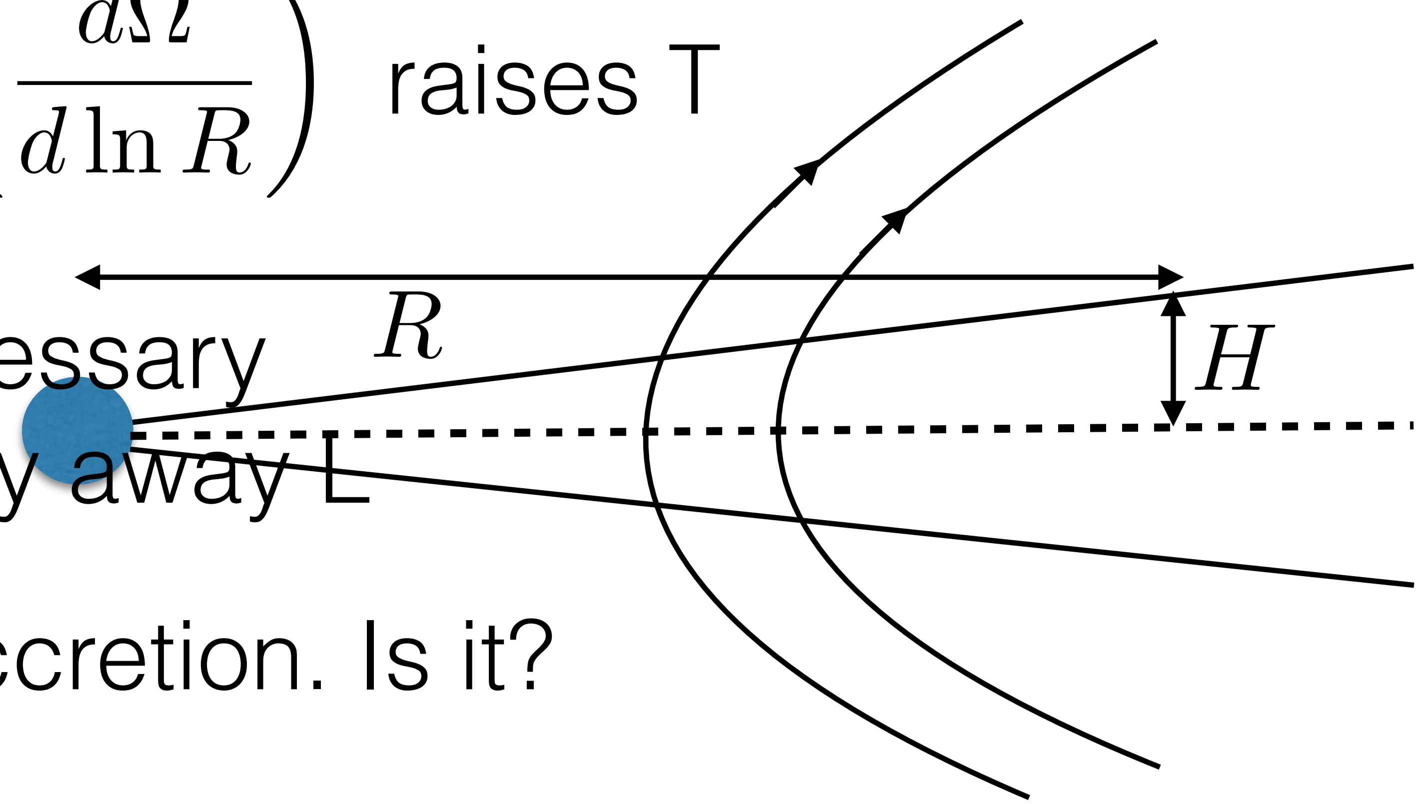
specific angular momentum $l = \sqrt{GMR} \propto R^{1/2}$

increases with R; since friction is an internal force
only small mass can carry away most angular momentum

$$\dot{M}_{\rm out} \sim \dot{M}_{\rm in} \left(\frac{R_{\rm in}}{R_{\rm out}} \right)^{1/2}$$

viscous heating $\rho \nu_{turb} \left(\frac{d\Omega}{d \ln R} \right)^2$ raises T

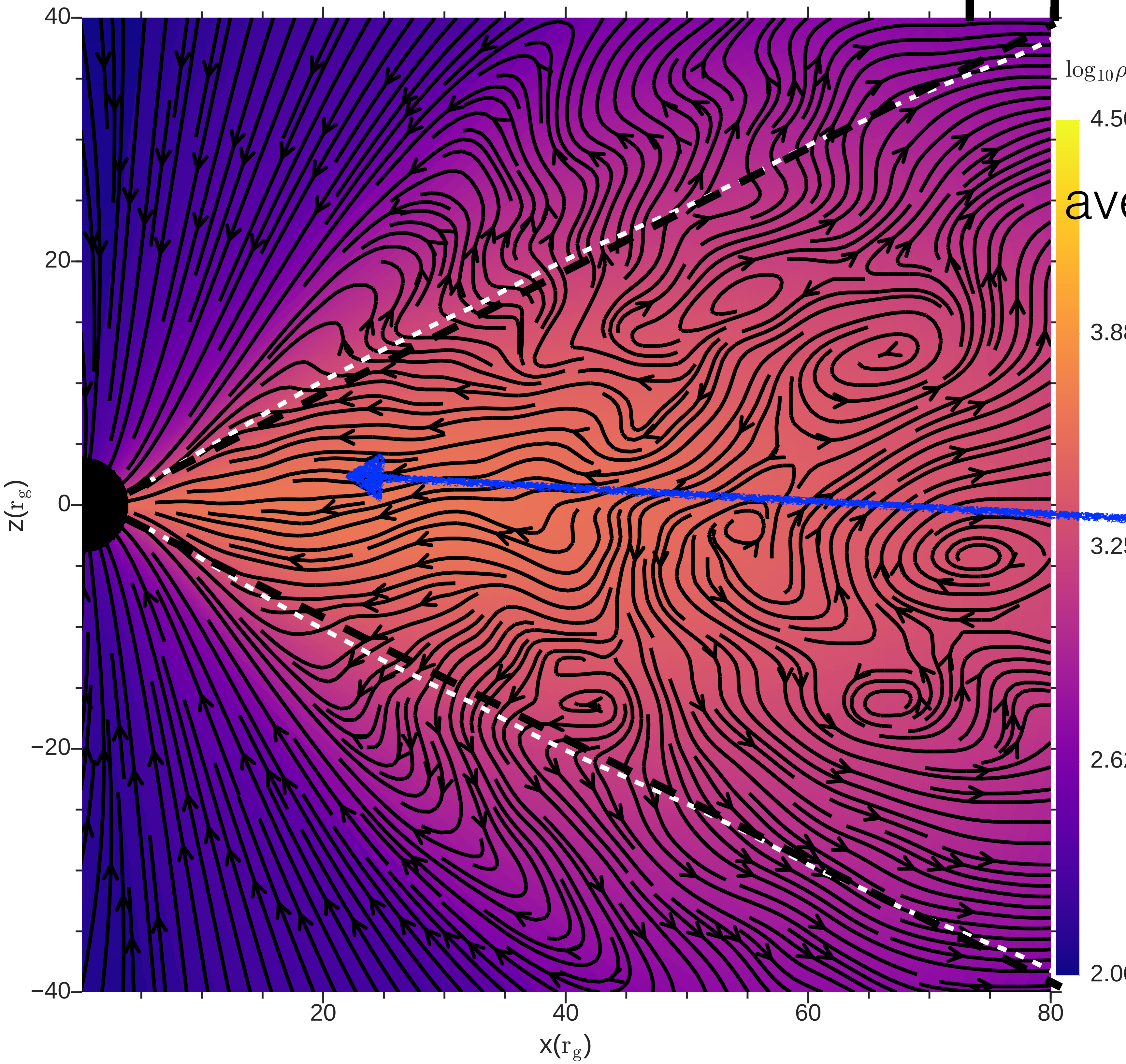
disk turbulence not necessary
magnetized wind can carry away L



MHD looks critical for accretion. Is it?

Mean field approach

àla Reynolds



average density w. poloidal velocity streamlines

mean flow caused by turbulent transport of ang. mom.

[image courtesy: Prasun Dhang]

disk temperature

$$T \sim 1\ \mathrm{keV} \left(\frac{M}{10\ M_\odot}\right)^{1/4} \left(\frac{\dot{M}}{10^{-8}\ M_\odot \mathrm{yr}^{-1}}\right)^{1/4} \left(\frac{r}{100\ \mathrm{km}}\right)^{-3/4}$$

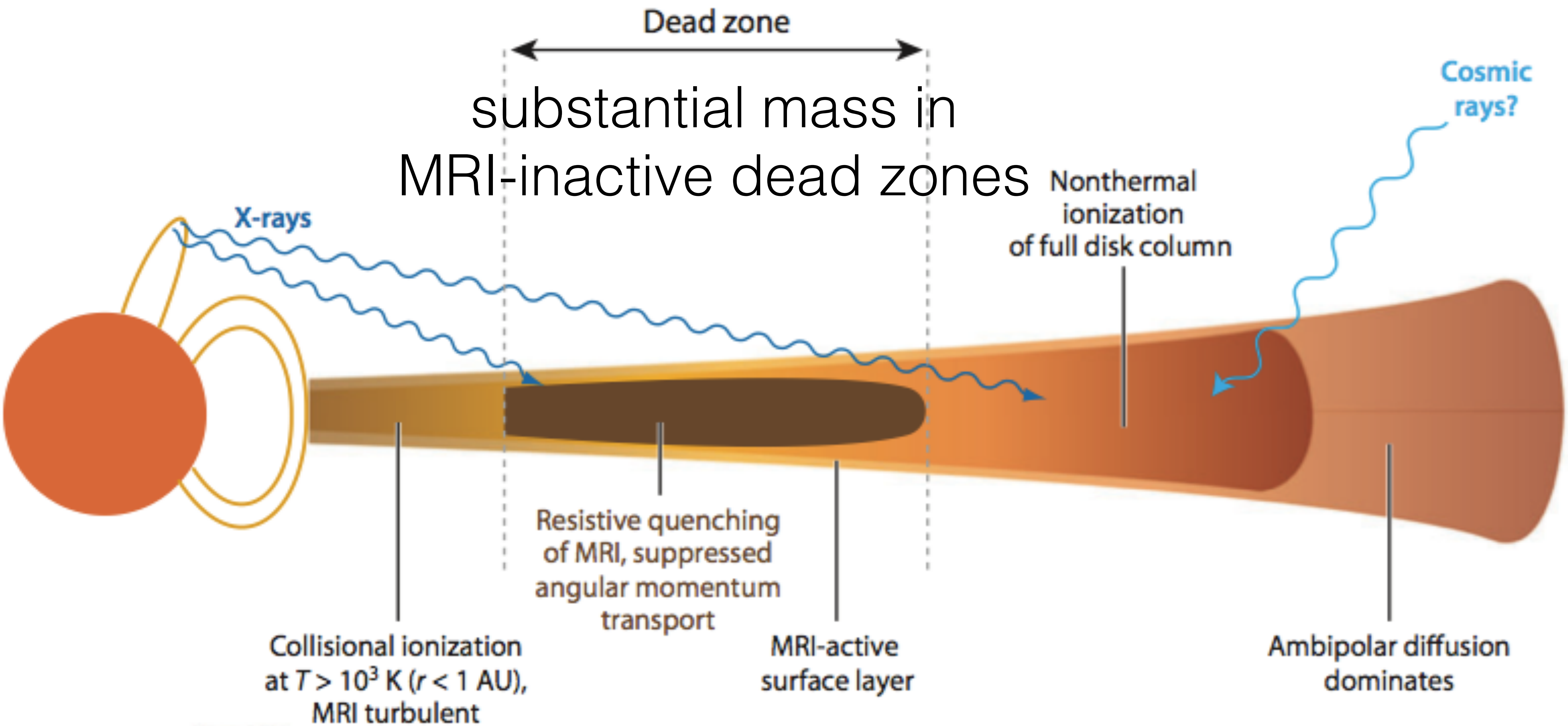
gives ~ 100 K for $1M_{sun}$ & r=1 AU
additional heating due to heating by stellar light

$$\frac{n_i n_e}{n_H} = \frac{(2\pi m_e k_B T)^{3/2}}{h^3} e^{-\chi/k_B T}$$

thermal ionization essentially gives x_e~0!
porto-planetary disks (PPDs) are completely neutral
non-thermal ionization due to X-ray,
CRs, radioactivity gives $x_e \sim 10^{-13}$

Is hydro turbulence possible?

substantial mass in
MRI-inactive dead zones

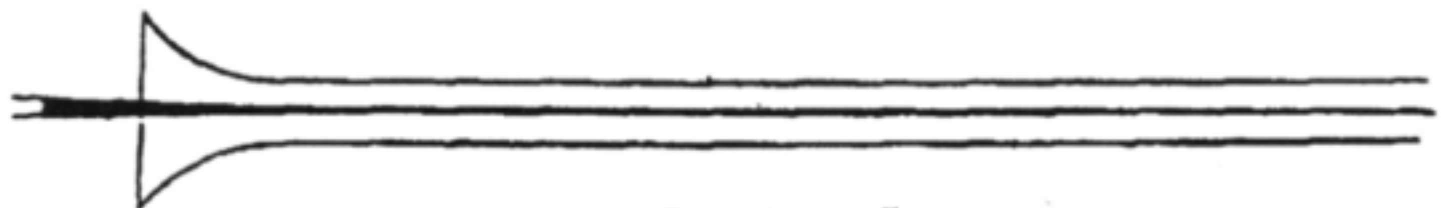


can hydro mechanisms operate?

various non-ideal MHD effects: resistivity, ambipolar diffusion, Hall term

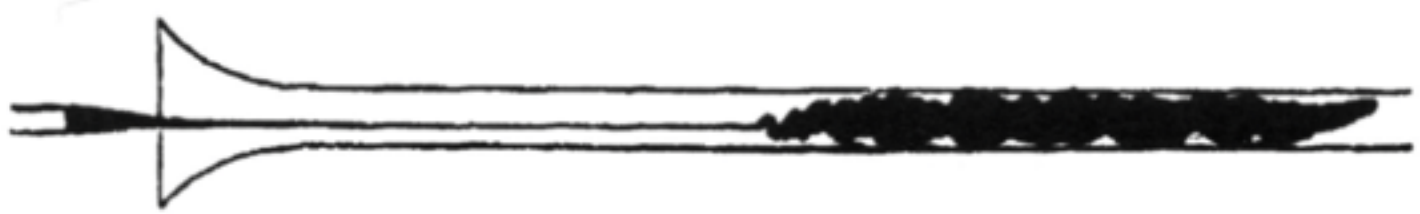
What abt hydro transport?

so common for laminar flow to break into turbulence at large Re



Laminar flow

pipe flow linearly stable for all Re; transition observed at Re~2000
sensitive to roughness, vibrations, etc.



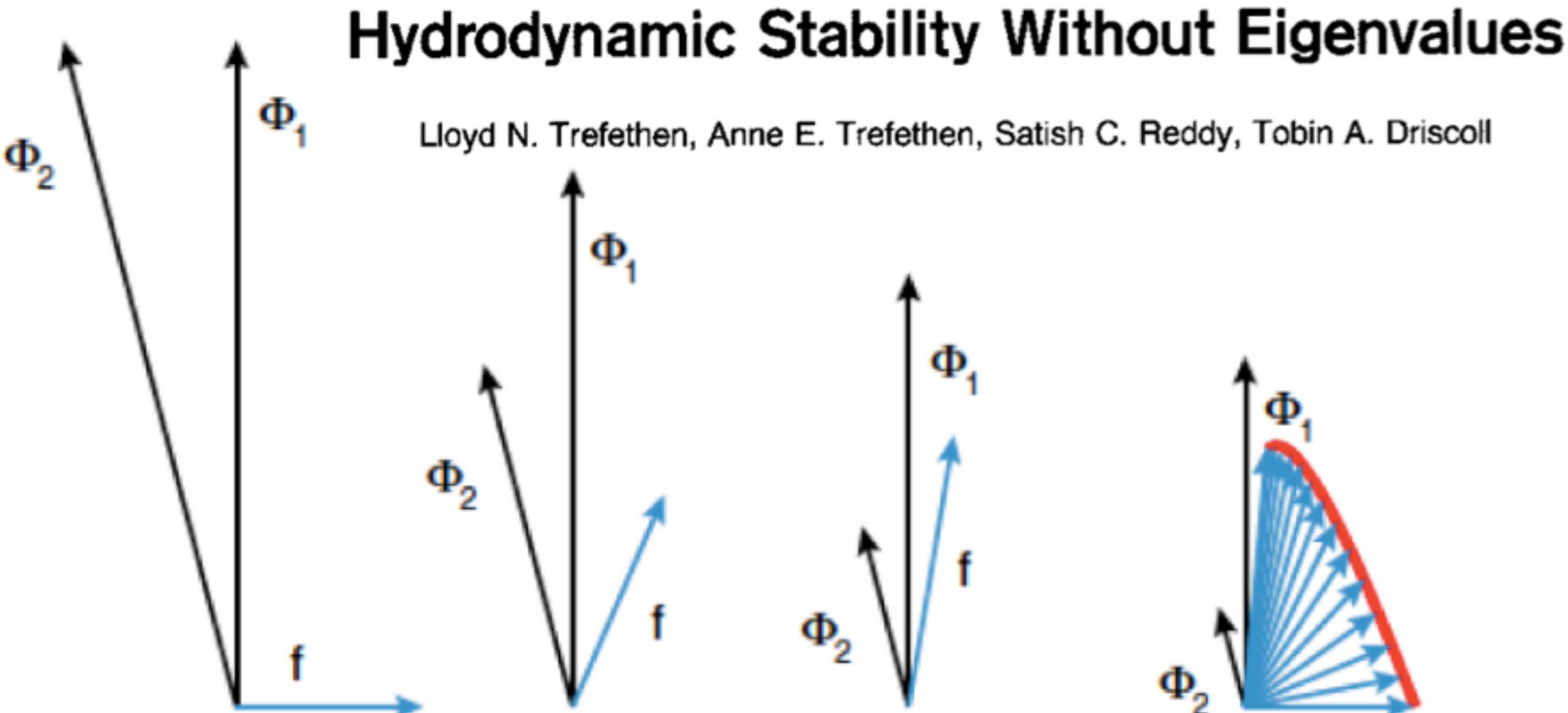
Turbulent flow

why shouldn't this happen in Keplerian flows where Re>10^{10}?

stabilizing role of (fast) epicyclic oscillations!

Various hydro ideas

transient growth due to non-normality of eigenvectors
transient growth by orders of magnitude
before eventual decay, nonlinearity may take over before this!



subcritical transition to turbulence

Other hydro instabilities

driven by vertical rotation gradient $\frac{\partial \Omega}{\partial z}$

angular momentum transport due to vertical convection

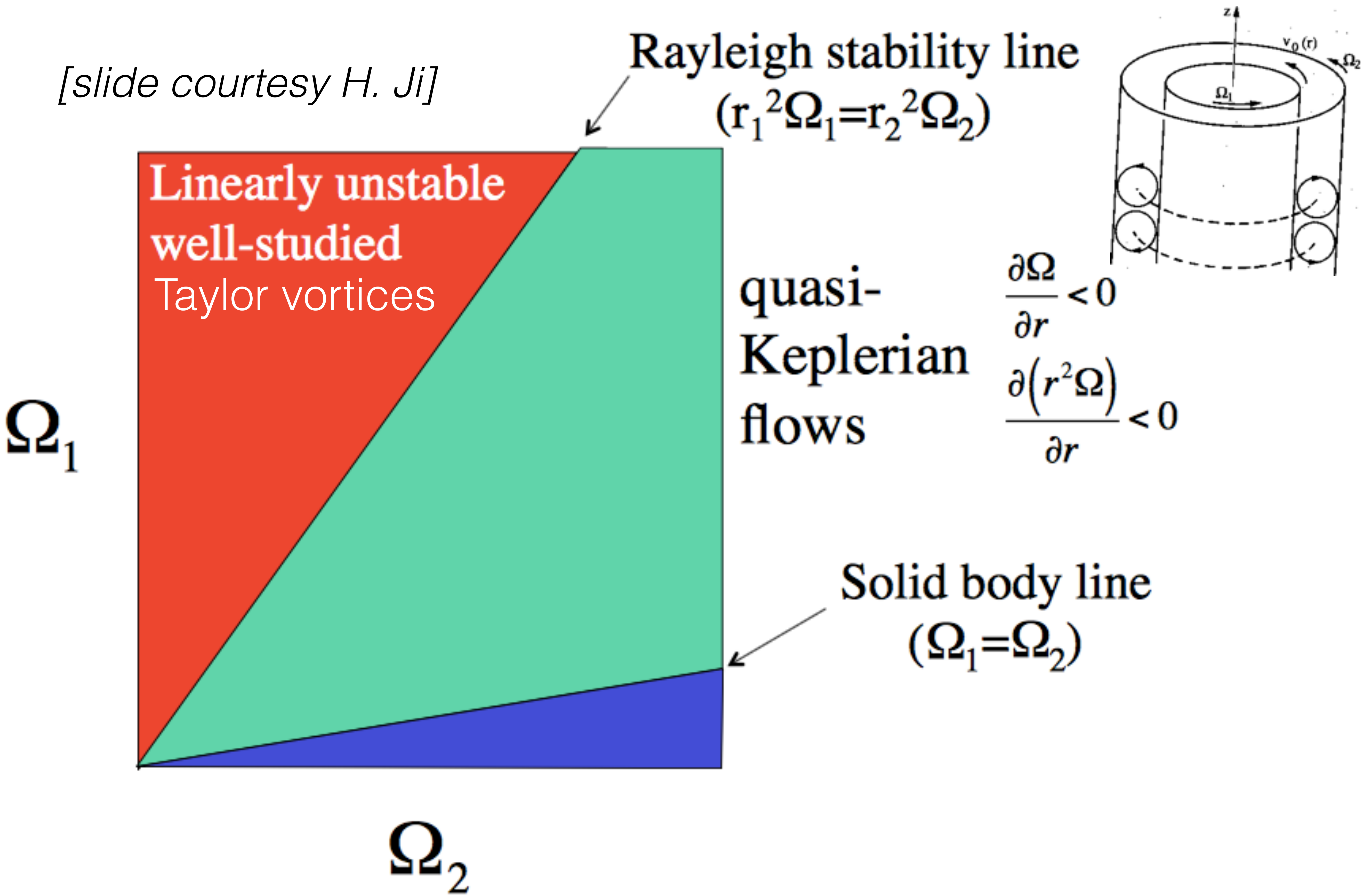
baroclinic instability driven by $\nabla p \times \nabla \rho$

Rossby-wave instability, …

none of hydro mechanisms unanimously accepted by community
BUT PPDs must accrete: layered accretion, MHD winds?
hard problem with lot of observations!

Taylor-Couette Experiments

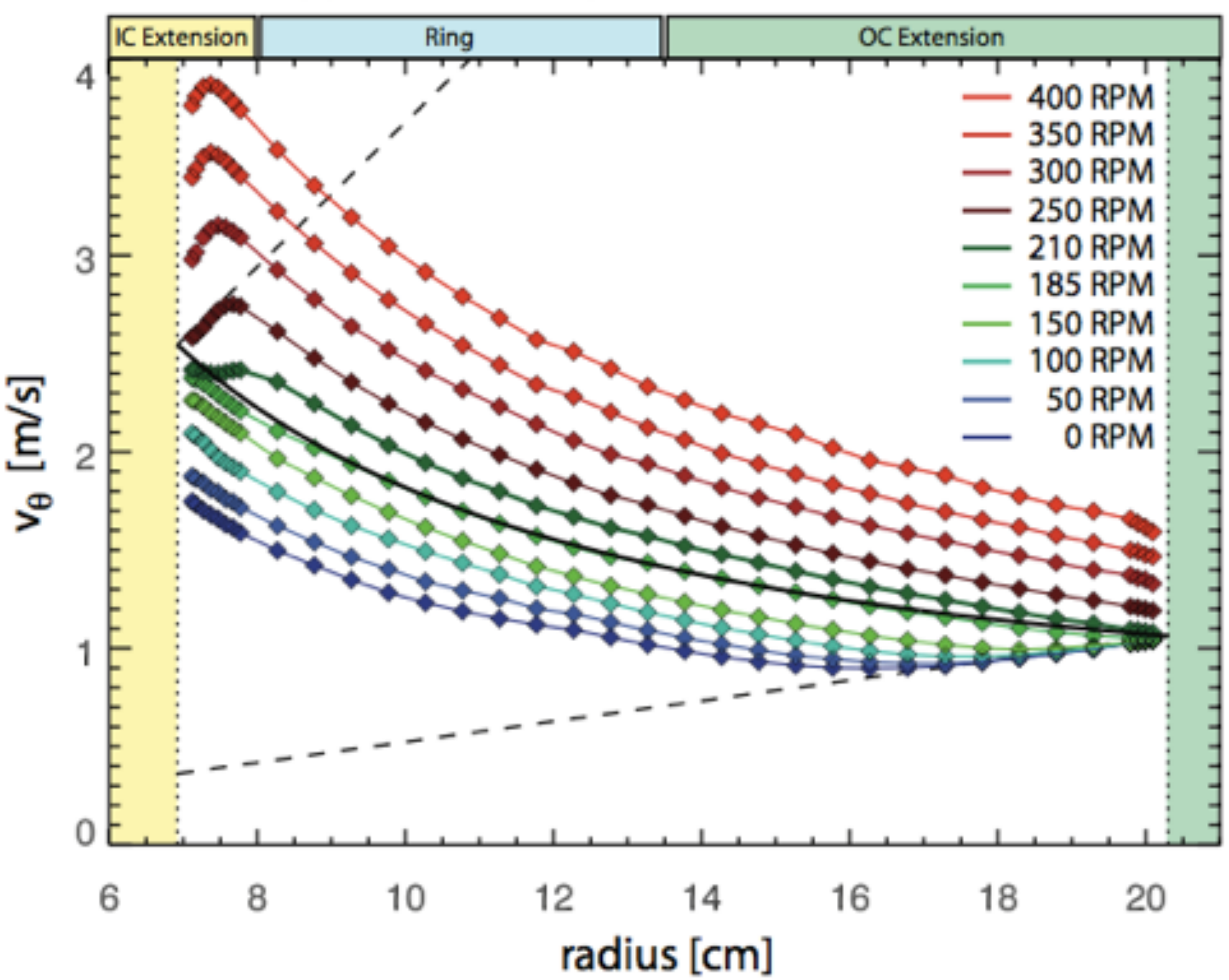
[slide courtesy H. Ji]



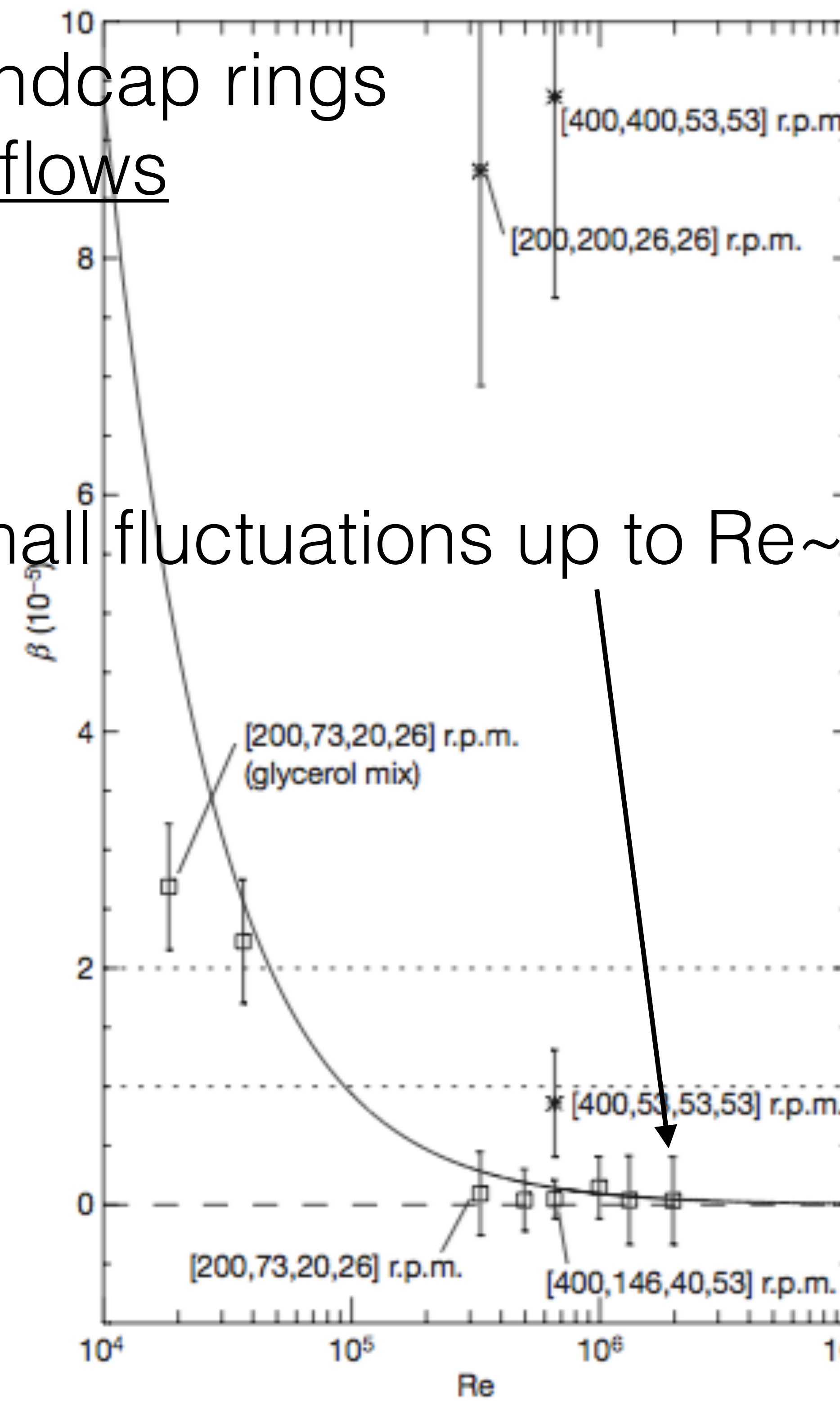
Experiments? [Ji et al. 2006]

independently rotating endcap rings
to prevent Ekman flows

quasi-Keplerian rotation

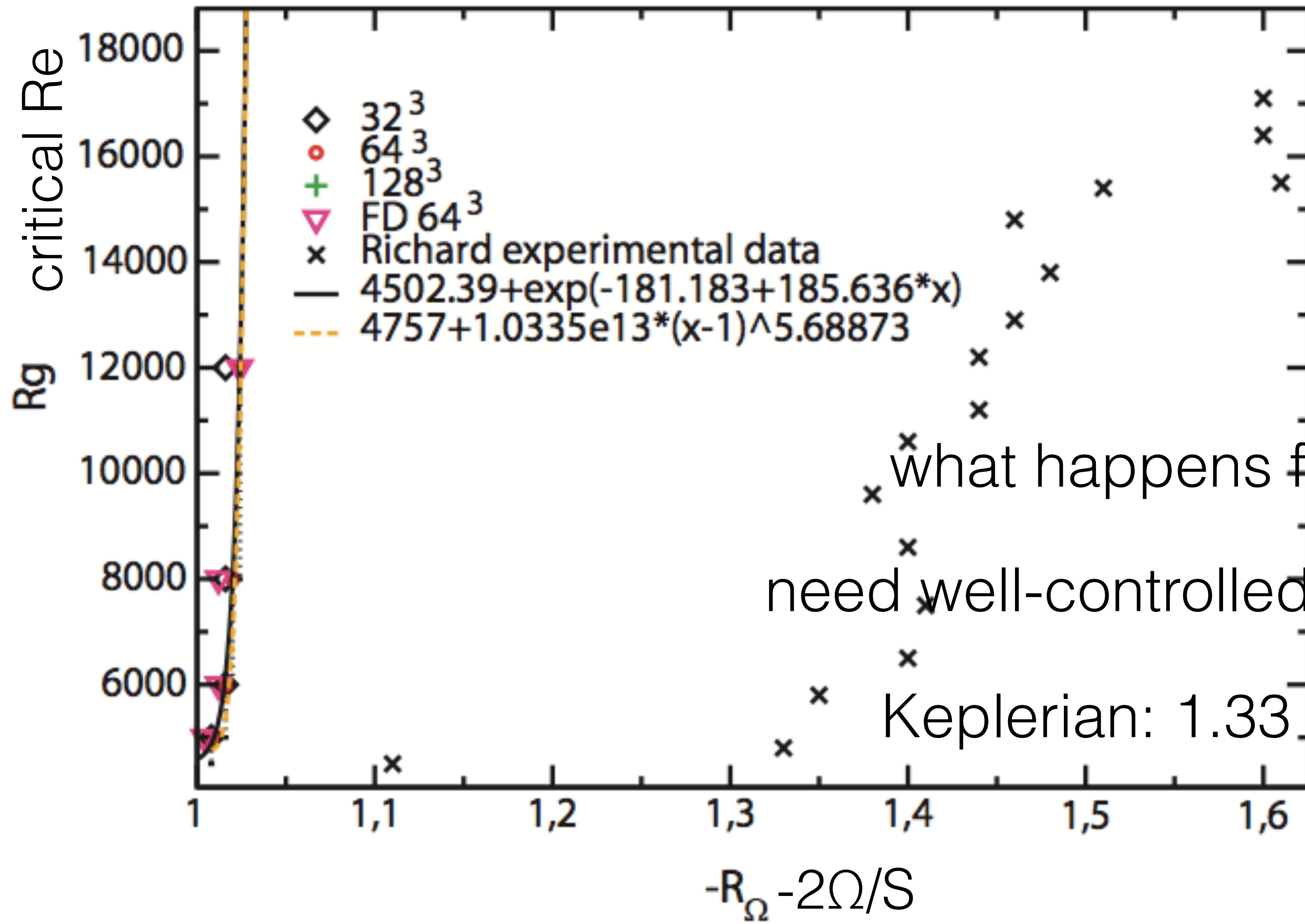


small fluctuations up to Re~10^6



Are hydro disks stable?

DNS confirm the role of Ekman boundary layers; *[Avila 2012]*



what happens for Re~10^{15}?

need well-controlled experiments

Keplerian: 1.33

Summary & Future

- angular momentum transport problem
- turbulent transport: MRI mechanism for ionized flow
- what about neutral disks? linear instabilities, nonlinear mechanisms, epicyclic stabilization
- Taylor-Couette flows at Re~10^{15}
- need experiments with controlled axial boundaries
- higher resolution local & global MHD sims with realistic microphysics (chemistry, thermodynamics, radiation, …)
- dynamo & self sustained B-fields; role of large scale fields & outflows

Thank you