

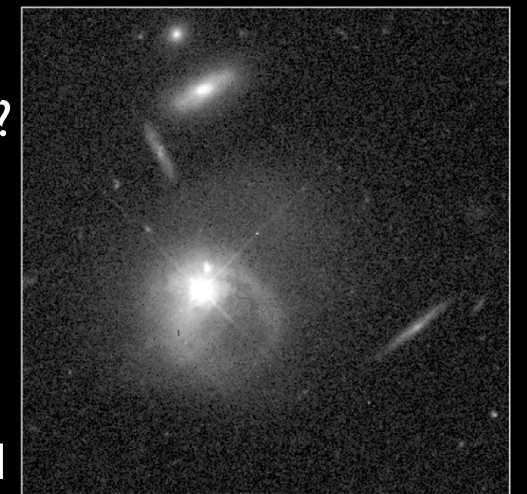
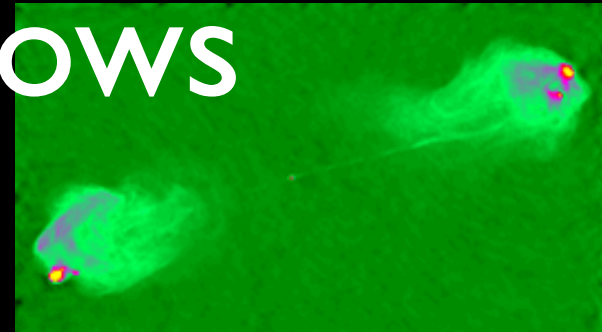


Heating & Angular Momentum Transport in Hot Accretion Flows

Prateek Sharma (UC Berkeley)
in collaboration with Eliot Quataert (UC Berkeley),
Greg Hammett, & Jim Stone (Princeton)

Low Luminosity Accretion Flows

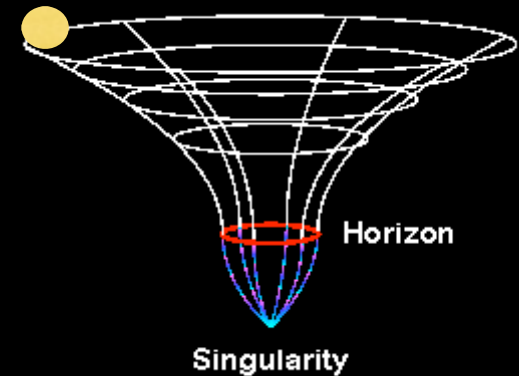
- 10^6 - $10^9 M_{\odot}$ BHs at centers of galaxies
- most luminous objects, e.g., quasars, AGN
- low luminosity BHs in nearby galaxies; why this dichotomy? may be there is just not enough material?
- $L = \eta \dot{M} c^2$; $\eta \sim 0.1$ for thin disks
- $\eta \sim 10^{-(\text{a few})}$ for LLBHs \Rightarrow disk hot & thick
- low η or low \dot{M} for low luminosity? requires detailed modeling



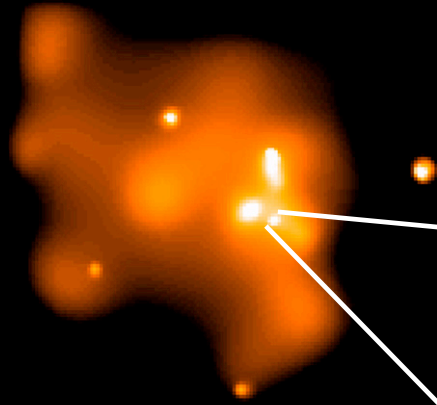
Quasar PKS 2349 HST · WFPC2
ST ScI OPO · January 1995 · J. Bahcall (Princeton), NASA

Radiative Efficiency

- energy at ∞ : 0
- energy for a stable orbit at r : $-GM/2r$
- rest $GM/2r$ goes into particle (thermal) energy; electron vs. ion? need detailed modeling to answer for collisionless plasmas
- in GR stable orbits only till $r > \text{few } r_s (\equiv 2GM/c^2)$;
 $E_{\text{rad}} \sim GM/2r_{\text{LSO}}$ if electrons heat and cool efficiently \Rightarrow
 $\eta \sim 0.1$
- $\eta \ll 0.1$ if electrons not heated efficiently



Sgr A*: Galactic Center BH



$4 \times 10^6 M_{\odot}$ black hole

$\dot{M} \sim 10^{-5} M_{\odot} / \text{yr}$ by stellar outflows

$L_{\text{obs}} \sim 10^{36} \text{ erg/s} \sim 10^{-5} \times (0.1 M_{\odot} c^2)$, radio to X-ray

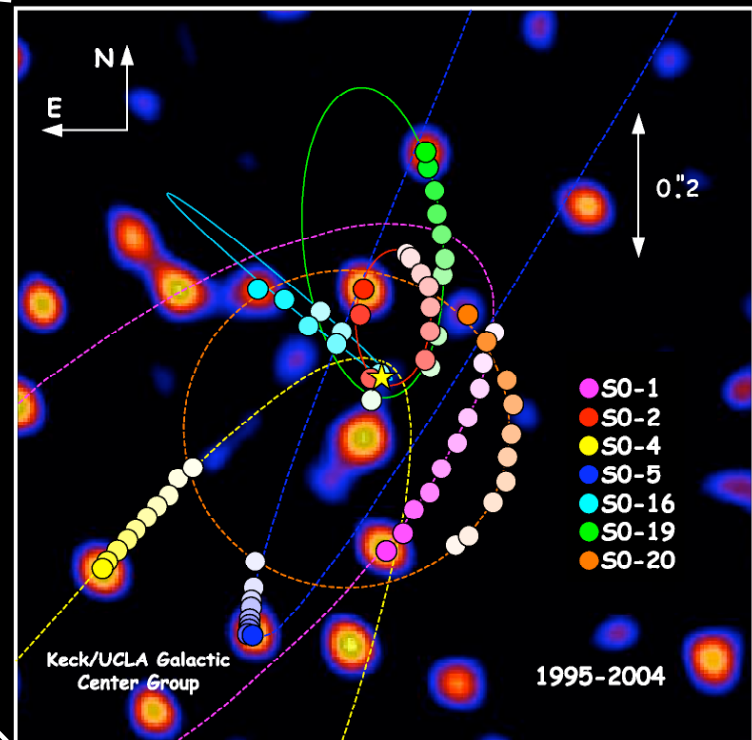
Why low luminosity? low \dot{M} or low η

outflows/convection can decrease \dot{M}

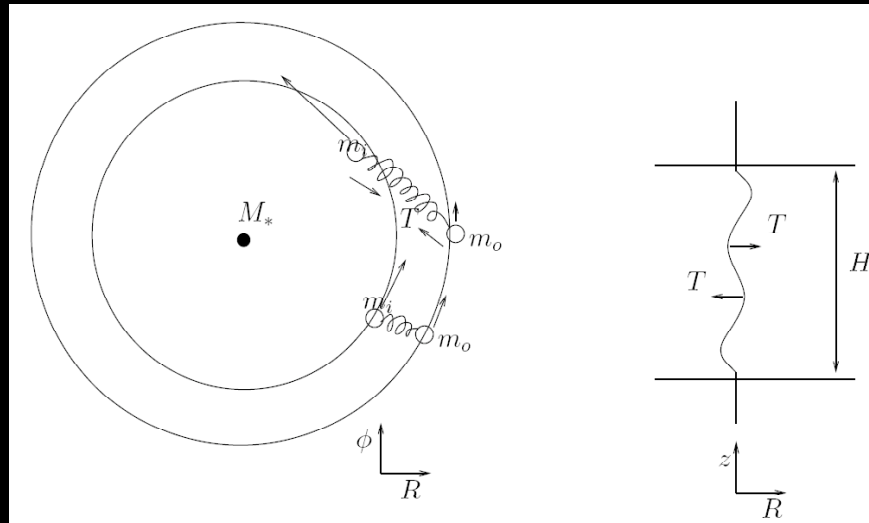
Bondi radius $\sim 0.07 \text{ pc}$ ($2''$), $n \sim 100/\text{cc}$, $T \sim 1.2 \text{ keV}$

[Baganoff et al. 2003]

$mfp \approx r_{\text{Bondi}}$, collisionless at smaller r ; detailed transport calculations useful



Disk Transport



Anisotropic

$$W_{r\phi} = - \left(1 - \frac{p_{\parallel} - p_{\perp}}{B^2} \right) \frac{B_r B_{\phi}}{4\pi} + \rho v_r \delta v_{\phi}$$

Maxwell

Reynolds

molecular viscosity not sufficient, invoke turbulent viscosity

Hydrodynamic disks linearly stable, magnetic fields qualitatively different

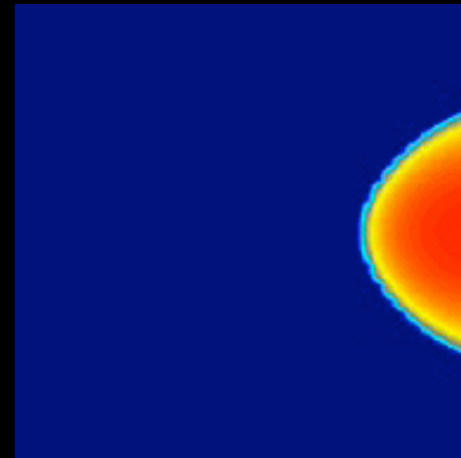
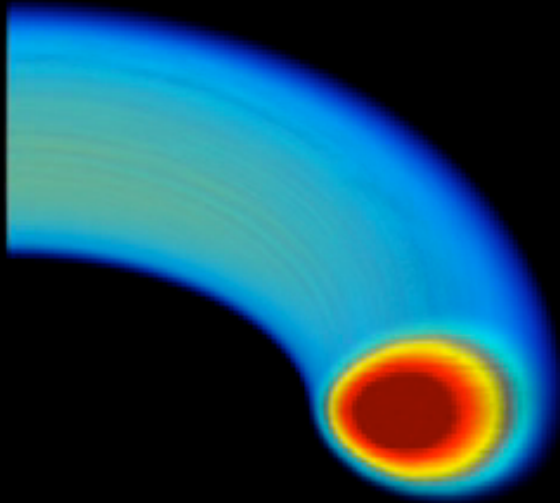
Source of turbulence is MRI when $d\Omega^2/d\ln r < 0$; r - ϕ correlations (due to shear) creates stress & causes transport

[Balbus & Hawley 1991]

Anisotropic viscous stress even if $B \rightarrow 0$ (cosmological implications); mass falls in & angular momentum flows out

3-D MHD Simulations

Movies by John Hawley



MHD simulations of MRI turbulence quite successful. Need to study it in collisionless regime applicable to Sgr A*

Drift Kinetic Equation

plasma is collisionless, hot w. $H \sim r$

Larmor radius \ll disk height

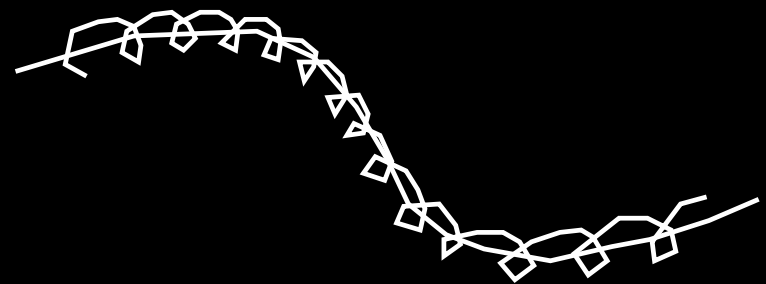
drift kinetic equation: approx. for Vlasov eq. if $k\rho_i \ll 1$, $\omega \ll \Omega_i$

Table 1.2: Plasma parameters for Sgr A*

Parameter	$r = r_{acc}$ 2.2×10^{17} cm	$r = \sqrt{r_{acc} R_S}$ 4.2×10^{14} cm	$r = R_S$ 7.8×10^{11} cm
$\nu_{i,ADAF}/\Omega_K \sim r^{3/2}$	11.4	9.4×10^{-4}	7.6×10^{-8}
$\nu_{i,CDAF}/\Omega_K \sim r^{3/2+p}$	11.4	1.81×10^{-6}	2.62×10^{-13}
$\rho_{i,ADAF}/H \sim r^{-1/4}$	2×10^{-11}	9.94×10^{-11}	4.59×10^{-10}
$\rho_{i,CDAF}/H \sim r^{-1/4-p/2}$	2×10^{-11}	2.23×10^{-9}	2.48×10^{-7}

$$\frac{\partial f_{0s}}{\partial t} + (\mathbf{V}_E + v_{\parallel} \hat{\mathbf{b}}) \cdot \nabla f_{0s} + \left(-\hat{\mathbf{b}} \cdot \frac{D\mathbf{V}_E}{Dt} - \mu \hat{\mathbf{b}} \cdot \nabla B + \frac{1}{m_s} (q_s E_{\parallel} + F_{g\parallel}) \right) \frac{\partial f_{0s}}{\partial v_{\parallel}} = 0$$

$\mu = v_{\perp}^2/B \propto T_{\perp}/B$ is conserved; $\mathbf{V}_E = c(\mathbf{E} \times \mathbf{B})/B^2$
 mfp \gg disk height scales \gg Larmor radius



Kinetic-MHD

Moments of the DKE

similar to MHD

pressure anisotropic wrt B

how p_{\parallel} , p_{\perp} evolve? next higher

order moment q_{\parallel} , q_{\perp}

closure problem; $q=0$ (CGL approx. may not be good)

$$\mathbf{q} \approx -n \nabla_{\parallel} T / (k_{\parallel} v_t + U)$$

[Snyder et al. 1997]

heat carried by free-streaming particles

captures collisionless effects like Landau damping

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) &= 0, \\ \rho \frac{\partial \mathbf{V}}{\partial t} + \rho (\mathbf{V} \cdot \nabla) \mathbf{V} &= \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi} - \nabla \cdot \mathbf{P} + \mathbf{F}_g, \\ \frac{\partial \mathbf{B}}{\partial t} &= \nabla \times (\mathbf{V} \times \mathbf{B}), \\ \mathbf{P} &= p_{\perp} \mathbf{I} + (p_{\parallel} - p_{\perp}) \hat{\mathbf{b}} \hat{\mathbf{b}}, \end{aligned}$$

$$\begin{aligned} \rho B \frac{D}{Dt} \left(\frac{p_{\perp}}{\rho B} \right) &= -\nabla \cdot \mathbf{q}_{\perp} - q_{\perp} \nabla \cdot \hat{\mathbf{b}}, \\ \frac{\rho^3}{B^2} \frac{D}{Dt} \left(\frac{p_{\parallel} B^2}{\rho^3} \right) &= -\nabla \cdot \mathbf{q}_{\parallel} + 2q_{\perp} \nabla \cdot \hat{\mathbf{b}}, \end{aligned}$$

Braginskii vs KMHD

- $(\delta p_{\parallel} - \delta p_{\perp})/p \sim 3\delta B/B - 2\delta\rho/\rho$: linear CGL limit
- $\Delta p/p \sim 1/\beta^{\alpha}$ ($\alpha \sim 0.5$) if Δp drives microinstabilities
- $\Delta p/p \sim (1/U) \mathbf{b} \cdot \nabla \mathbf{u}$; a shear stress in Braginskii
- pressure anisotropy is like parallel viscosity
- viscous heating rate: $\Delta p (\mathbf{b} \cdot \nabla \mathbf{u})$
- $q_e/q_i \sim (T_e/T_i)^{1-\alpha}$ for KMHD
- $q_e/q_i \sim (T_e/T_i)(v_i/v_e) \sim (m_e/m_i)^{1/2}(T_i/T_e)^{5/2}$ in Braginskii

Anisotropic transport

Pressure anisotropy equivalent to anisotropic viscous stress, in addition to Reynolds & Maxwell stresses

$$\frac{\partial}{\partial t}(\rho V) + \nabla \cdot \left(\rho V V + \left(p_{\perp} + \frac{B^2}{8\pi} \right) I - \frac{B B}{4\pi} \left(1 - \frac{p_{\parallel} - p_{\perp}}{B^2} \right) \right) = 0$$

Large scale anisotropic viscous heating, small-scale resistive, viscous heating

$$\frac{\partial}{\partial t} e + \nabla \cdot (e V + q) = -p_{\perp} \nabla \cdot V - (p_{\parallel} - p_{\perp}) b b : \nabla V + \eta_R j^2 + \eta_V |\nabla V|^2$$

$$\delta p_{1s} = -\frac{p_{0s}}{v_s} (3 \hat{b} \cdot \nabla U \cdot \hat{b} - \nabla \cdot U)$$

$$\delta p = p_{\parallel} - p_{\perp}$$

In Braginskii regime, $U \gg kv_t$, pressure anisotropy reduced by Coulomb collisions

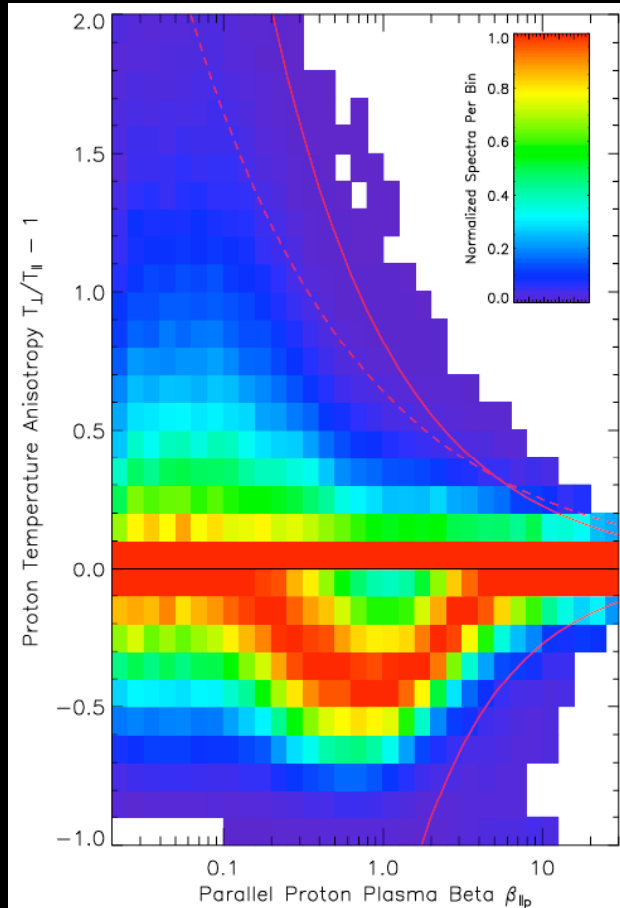
For $U \ll kv_t$ anisotropy governed by μ invariance

Can anisotropy be arbitrarily large? No.

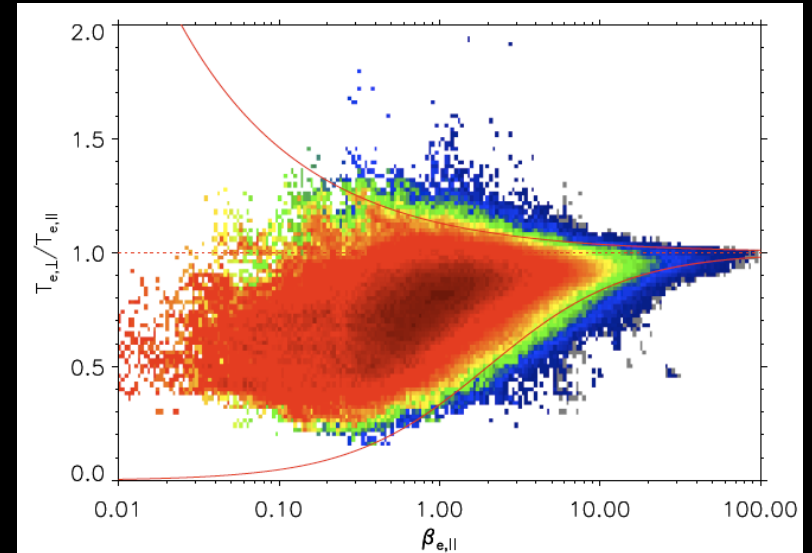
Δp limits

Protons; [Kasper et al. 2003]

Electrons; [S. Bale]



$$\left| \frac{p_{\perp}}{p_{\parallel}} - 1 \right| \leq \frac{S}{\beta^{\alpha}}$$



Pressure anisotropy reduced by Larmor-scale instabilities:

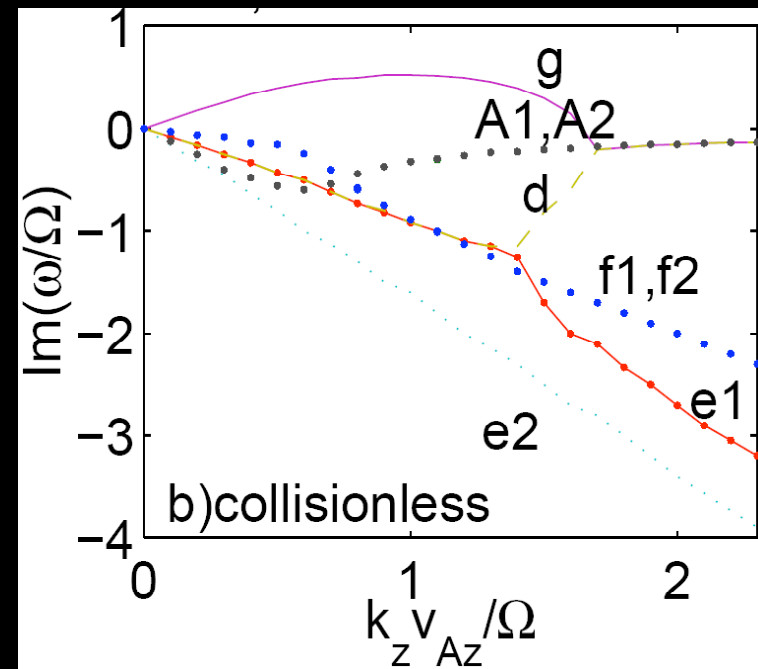
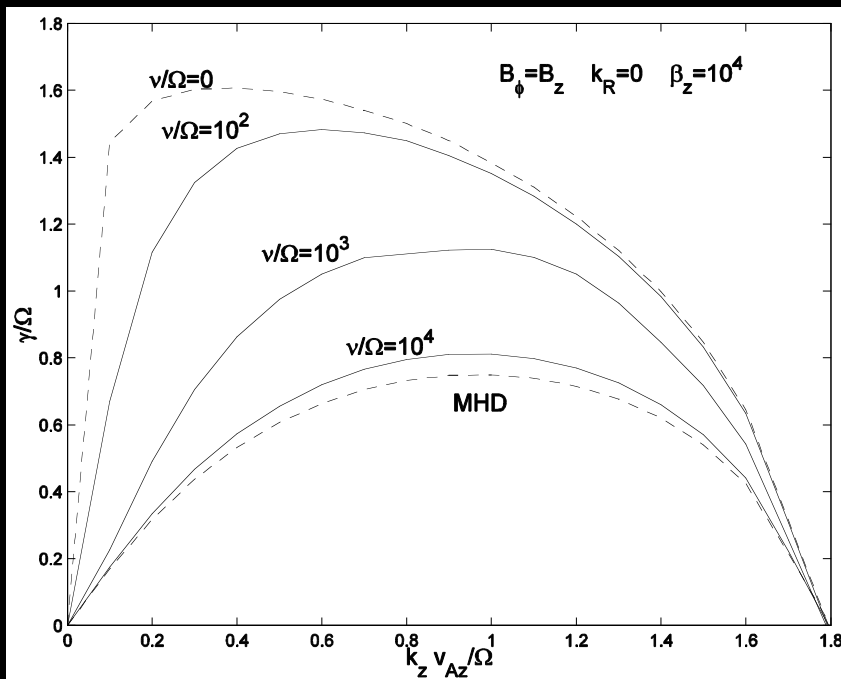
protons: ion-cyclotron, mirror ($p_{\perp} > p_{\parallel}$)

electrons: electron-whistler ($p_{\perp} > p_{\parallel}$)

firehose for ($p_{\perp} < p_{\parallel}$)

agree with kinetic PIC simulations [Gary et al.]

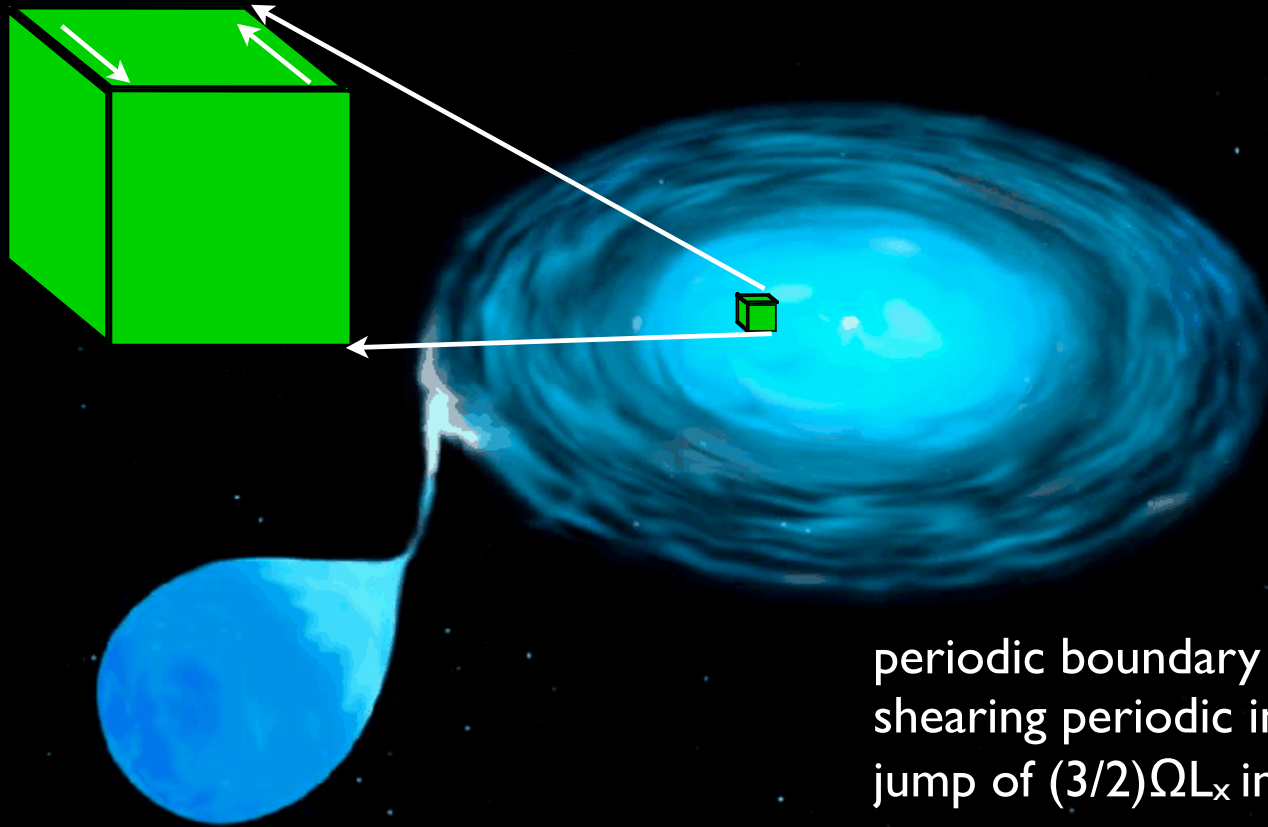
Collisionless MRI



fastest growing mode twice faster than in MHD, at much larger scales

collisionless damping, large scale dissipation $dv_{\parallel}/dt = -\mu \nabla_{\parallel} B + eE_{\parallel}/m$
 [Quataert et al. 2002; Sharma et al. 2003; Balbus 2004]

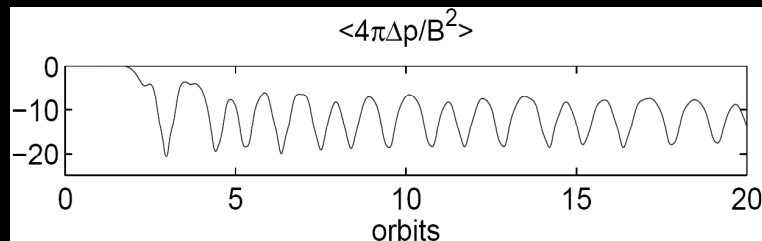
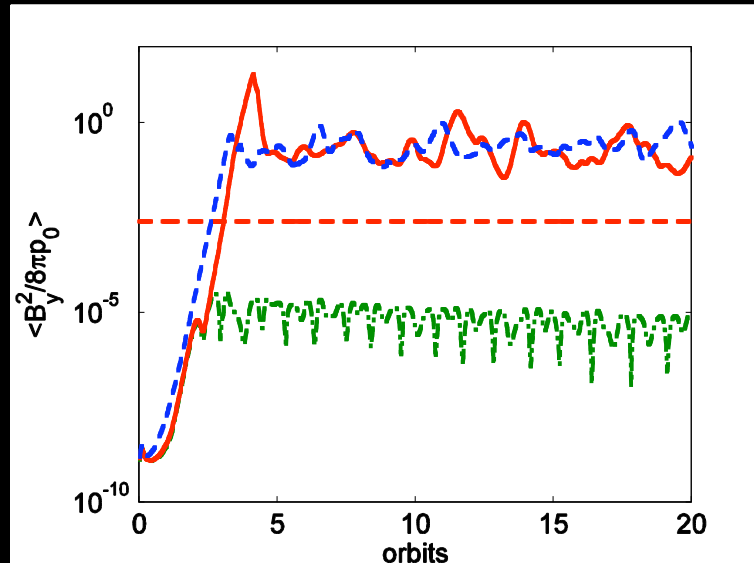
Shearing-box sims.



periodic boundary conditions in ϕ, z
shearing periodic in r
jump of $(3/2)\Omega L_x$ in V_ϕ

analogous to flux tube sims. in fusion
where shear is in B

Δp due to MRI



$$B \cdot \nabla B \longrightarrow \left(1 - \frac{(p_{\parallel} - p_{\perp})}{B^2} \right) B \cdot \nabla B$$

pressure anisotropy ($p_{\perp} > p_{\parallel}$) as $B \uparrow$

$$\mu \propto \langle v_{\perp}^2 \rangle / B \propto p_{\perp} / B = \text{const.}$$

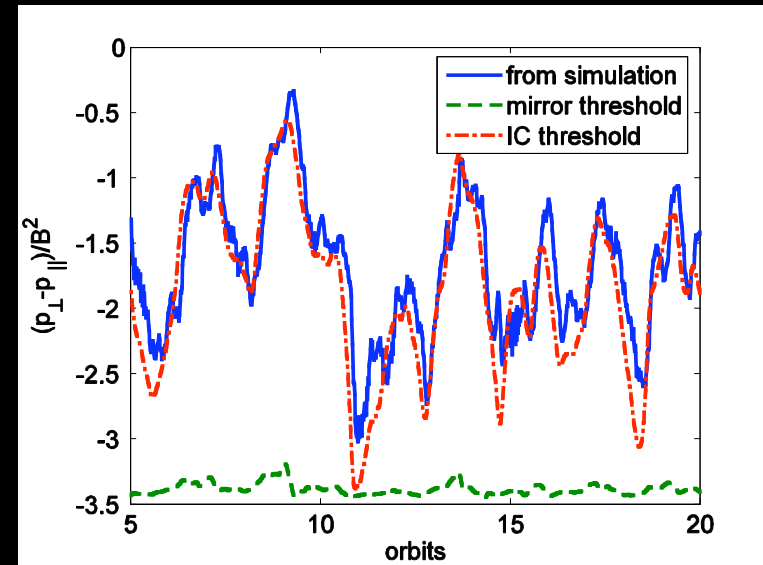
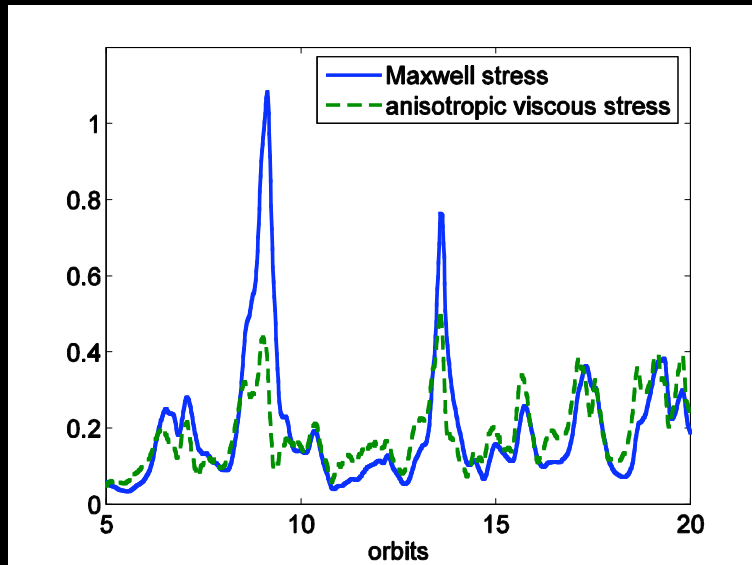
pressure anisotropy can stabilize MRI modes

How large can pressure anisotropy become? Anisotropy driven instabilities: mirror, ion cyclotron, etc.

$$\Delta p / p \approx O(1) / \beta, \quad \beta = 8\pi p / B^2 \sim 1 - 100$$

Microinstabilities \Rightarrow MHD like dynamics

Pressure anisotropy



anisotropic stress \sim Maxwell stress (can dominate at $\beta \gg 1$)

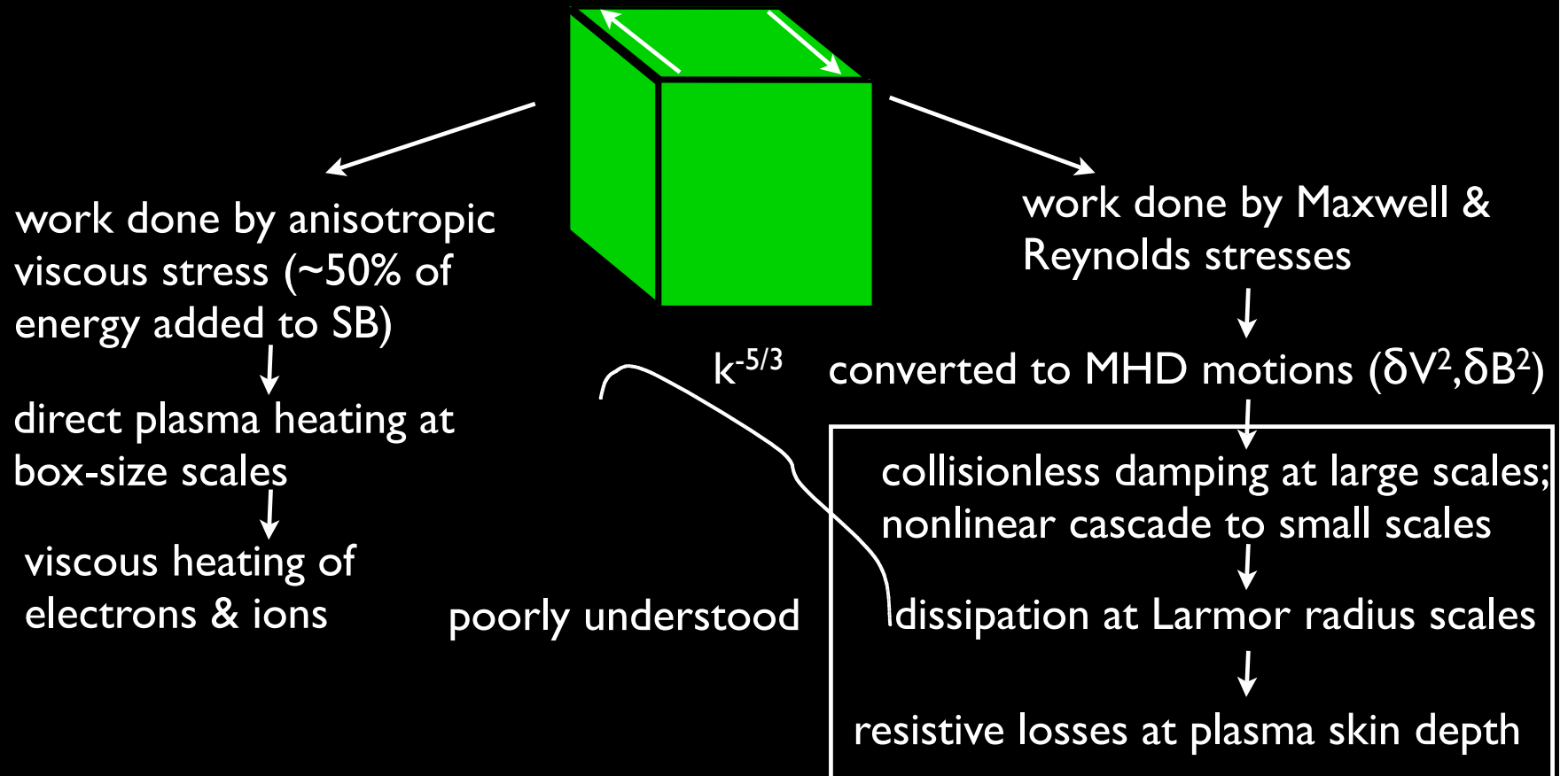
anisotropic pressure \Rightarrow 'viscous' heating (due to anisotropic stress) at large scales

ion pressure anisotropy limited by IC instability threshold

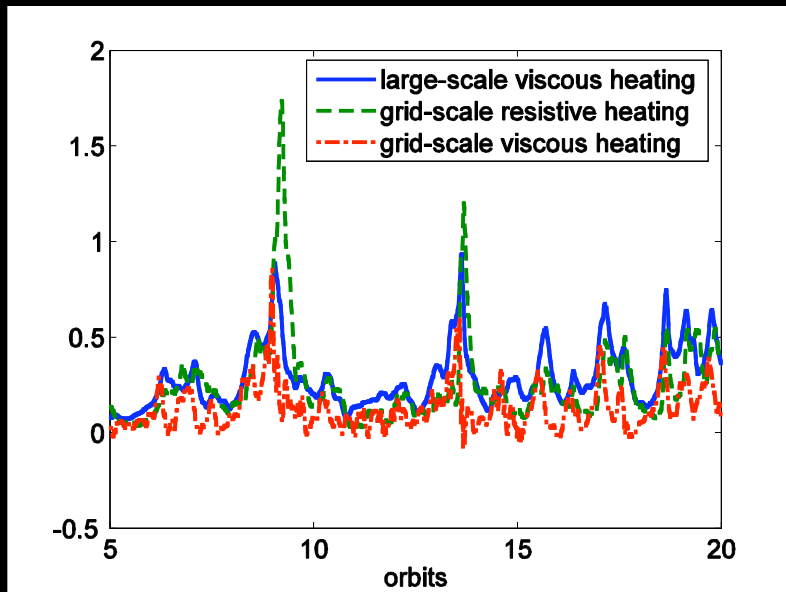
Will electrons also be anisotropic? Yes, collision freq. is really tiny

electron pressure anisotropy reduced by electron whistler instability

Shearing-box energetics



Electron heating



In sims. anisotropic heating numerical losses => half the energy is captured as heating due to anisotropic pressure

Form of pressure anisotropy threshold from full kinetic theory for both electrons & ions:

$$\frac{p_{\perp}}{p_{\parallel}} - 1 = \frac{S}{\beta^{\alpha}}$$

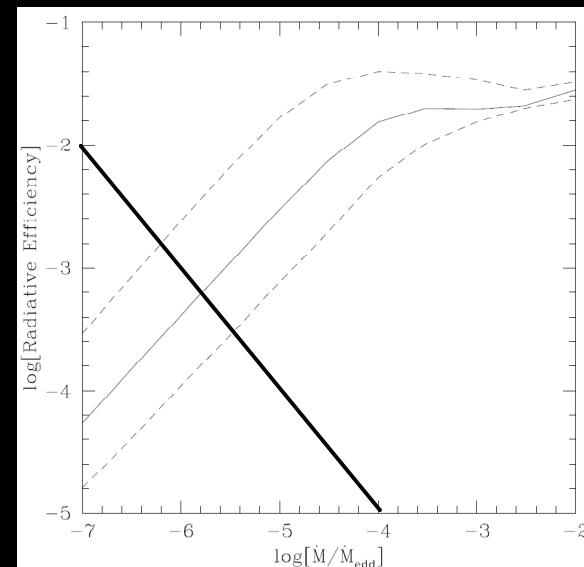
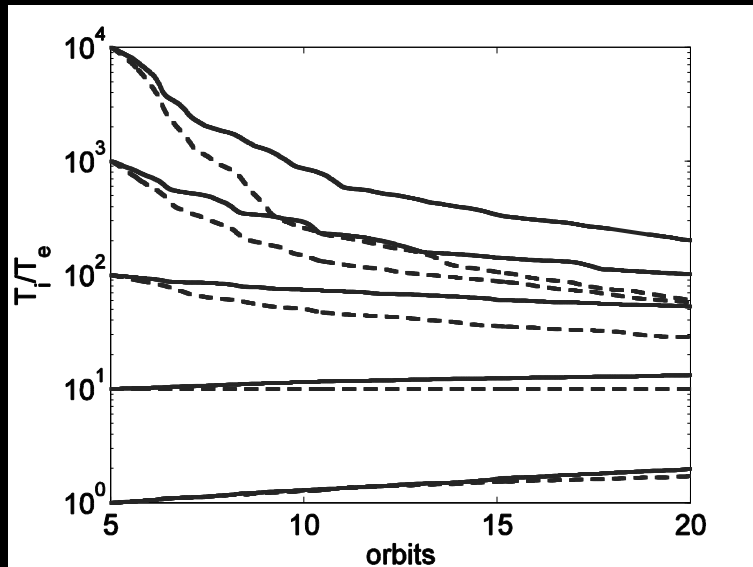
Ratio of electron & proton heating rates

$\alpha \sim 0.5$, $S_e \sim 0.4 S_i$ for ion cyclotron/electron whistler instabilities
=> significant electron heating (compare with Braginskii where ions are heated preferentially)

$$\frac{q_e}{q_i} = \frac{\Delta p_e}{\Delta p_i} \sim \left(\frac{T_e}{T_i} \right)^{1/2}$$

Results depend on pitch angle scattering thresholds (which are fairly well-tested)

Radiative efficiency



Even if electrons are cold initially, viscous heating will eventually give $T_e/T_i \sim 1$ (few 10s), neglecting synchrotron cooling of electrons

measured electron temperature $\sim 3 \times 10^{10}$ at $\sim 24 r_s$ [Bower et al. 2004]

Electrons somewhat radiatively efficient w. $\eta \sim 10^{-3}$ & $\dot{M} \sim 10^{-7} \dot{M}_{\text{D}}$ /yr consistent with Faraday RM observations & global MHD sims.

Conclusions

- pressure anisotropy natural as μ conserved
- scattering due to microinstabilities
- anisotropic stress \approx Maxwell stress
- significant e^- heating \Rightarrow radiative (ADAF w. $\eta \sim 10^{-5}$ ruled out)
- $\dot{M} \ll \dot{M}_{\text{Bondi}}$ for low luminosity; consistent with rotation measure toward Sgr A*