TERM PAPER

1) **Solution of Landau level problem in a tight-binding model. This is also known as Hofstadter butterfly model or TKNN theory in tight-binding model with slight modification.**

Imagine a 2D square lattice in the xy-plane, and a magnetic field penetrating perpendicular to it (i.e. **B** = B_z). The electrons are not interacting. We have solved this problem in the continuum limit, assuming kinetic energy term as H = $p^2/2m$, where p is momentum and m is the mass.

(a) *Hofstadter butterfly model. Now solve the Landau level problem with tight-binding method. Plot density of states as a function of magnetic field and chemical potential*

(b) *What is TKNN theory*

Refs: i) D. J. Thouless, M. Kohmoto, M. P. Nightingale, and M. den Nijs
Phys. Rev. Lett. 49, 405 – Published 9 August 1982
ii) http://www.physics.iisc.ernet.in/~qcmjc/talk_slides/QCMJC.2013.08.22_Yinghai.pdf

2) **Solve edge state problem in 2D/3D.**

(a) Solve the continuum model for 2D and 3D for any of your favorite model

(b) Repeat the calculation in a finite lattice tight-binding model with open boundary condition in any one direction. Plot the edge and quantum well states and demonstrate bulk-boundary correspondence.

(c) Also solve the problem with Transfer matric method.

References:

i) Bernevig's book, Chapter 6. Sec 6.2, S. Q. Shen's book.
ii) B. Andrei Bernevig, Taylor L. Hughes, Shou-Cheng Zhang, Science 314, 1757 (2006)
iii) X.-L. Qi, S. C. Zhang, RMP 83, 1051 (2011)

3) **Topological crystalline insulator (TCI)**

a) Discuss what is TCI.
b) Reproduce the derivation of Z2 invariant for TCI
c) Either solve for the Z2 invariant or Mirror Chern number for a model TCI Hamiltonian

References:

i) Liang Fu, Phys. Rev. Lett. 106, 106802 – Published 8 March 2011
ii) Timothy H. Hsieh, Nature Communications volume3, Article number: 982 (2012)
iii) Yoichi Ando, Liang Fu, arXiv:1501.00531.

4) **Axion insulators**

a) What is axion insulator
b) Derive Eq. 3 of Ref. i below, or Eq. 3 of Ref. ii. (Same equation).
c) Plot axion angle as a function of magnetic moment for a different Hamiltonian that I will give you.

References

i) Rundong Li, Nature Physics 6, 284 (2010)
ii) Akihiko Sekine, and Kentaro Nomura, arXiv:1401.4523
iii) Jing Wang, et at PRL 106, 126403 (2011)

5) **Topological Kondo Insulator**

a) What is Kondo and mixed valence insulator?
b) How topological phase transition occurs in a Kondo insulator
c) Properties of edge state
References: (a) Maxim Dzero, Jing Xia, Victor Galitski, Piers Coleman, arXiv:1506.05635
(b) Maxim Dzero, Kai Sun, Victor Galitski, Piers Coleman, PRL 104, 106408 (2010)
(c) Maxim Dzero, Kai Sun, Piers Coleman, Victor Galitski, PRB 85, 045130 (2012)
(d) Victor Alexandrov, Maxim Dzero, Piers Coleman, PRL 111, 226403, (2013)

6) **Topological superconductor**

Kitaev model is a topological superconductor in 1D

a) Give examples of topological superconductors for s-wave and p-wave pairings
b) How to obtain the topological invariant for such a system
c) What is the edge mode for topological superconductor?

References: i) Bernevig's book, Chapter 6. Sec 6.2, S. Q. Shen's book.
ii) X.-L. Qi, S. C. Zhang, RMP 83, 1051 (2011)
iii) Masatoshi Sato, Yoichi Ando, Rep. Prog. Phys. 80, 076501 (2017); arXiv:1608.03395
iv) Xiao-Liang Qi, et al, PRL 102, 187001 (2009)
v) Fan Zhang, C. L. Kane, and E. J. Mele, PRL 111, 056402 (2013)
vi) Fan Zhang, C. L. Kane, and E. J. Mele, PRL 111, 056403 (2013)

7) Floquet topological insulator

a) What is Floquet Hamiltonian?
b) How non-trivial Berry phase arise in Floquet Hamiltonian
c) Demonstrate the result with at least one example case.
References: (i) N H. Lindner, Gil Refael & V Galitski, Nature Physics volume 7, pages 490–495 (2011)
(ii) Jérôme Cayssol, Balázs Dóra, Ferenc Simon, Roderich Moessner, arXiv:1211.5623

8) **3D Dirac and Weyl semimetals**

a) What is a 3D Dirac Hamiltonian and how they disintegrate into Weyl fermions
b) Demonstrate the topological properties for these cases with at least one example case
c) Fermi arc on the surface of Weyl fermions
References: (i) N.P. Armitage, E. J. Mele, Ashvin Vishwanath, Rev. Mod. Phys. 90, 15001 (2018)
(ii) Oskar Vafek, Ashvin Vishwanath, arXiv:1306.2272
(iii) Gábor B. Halász, Leon Balents, Phys. Rev. B 85, 035103 (2012)
(iv) A.A. Burkov, arXiv:1704.06660

9) **Pseudo-magnetic field, Landau level in graphene/Dirac/Weyl cones without external magnetic field**

Due to chemical potential gradient, strain etc, graphene and similar Dirac system can obtain a pseudo magnetic field.

(a) Show the derivation
(b) Estimate the magnetic field strength as a function of strain or chemical potential gradient
(c) Show how Landau level forms

References: (i) F. Guinea, M. I. Katsnelson, A. K. Geim, Nature Phys.6:30-33,2010
(ii) Hassan Shapourian, Taylor L. Hughes, and Shinsei Ryu, PRB 92, 165131 (2015), Sec II
(iii) Alberto Cortijo, et al PRL 115, 177202 (2015)
(iv) Hiroaki Sumiyoshi and Satoshi Fujimoto, Phys. Rev. Lett. 116, 166601 (2016)

10) **Berry phase in three band model and SU(3) symmetry**

a) Write a generic three band model in terms of Gell-Mann matrices

b) Derive the expression for Berry curvature and Chern number for three band model

c) For the Hamiltonian given in Ref. (i) below, derive analytically the Berry curvature and berry phase

References: (i) S. Ray, A. Ghatak, T Das, Physical Review B 95, 165425 (2017)

(ii) R. Barnett, G. R. Boyd, and V. Galitski, Phys. Rev. Lett. 109, 235308 (2012).

11) **Altshuler, Aronov, Spivak oscillations**

(a) Show that for a network of Aharonov-Bohm rings in parallel or in series, the oscillation period is h/2e.

12) **Derive the formula for how berry phase modifies the Shubnikov-de Haas van Alphen oscillations and Landau level. In other words, how to obtain the value of Berry phase from these measurements**

13) **Show that a system with finite Berry phase (such as graphene or Dirac/Weyl semimetals) give negative magnetoresistance.**

14) **Counter-propagating edge channels in fractional quantum Hall regime -** under certain circumstances, the edge channels in the FQH regime can be counter-propagating (i.e. not all of them on a given edge move in the same direction).

a) Discuss the conditions that can lead to this situation
b) What will be the implications of this on the conductivity of the system? Discuss with a specific example.