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Letter to the Editor

The solar dynamo with meridional circulation

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Abstract. We show that meridional circulation can have a profound influence on dynamo models for the solar cycle. Motivated by the observed tilt angles of sunspot groups we assume that the generation of the poloidal field takes place near the surface, while a shear layer of radial differential rotation produces the toroidal field at the bottom of the convection zone. Both layers are coupled by a circulation with a poleward directed flow in the upper part and an equatorward flow in the deep layers of the convection zone. The circulation forces the toroidal field belts (which are responsible for the surface activity) to move equatorward. This leads to butterfly diagrams in qualitative agreement with the observations, even if the dynamo wave would propagate poleward in the absence of circulation. This result opens the possibility to construct models for the solar cycle which are based on observational data (tilt angles, differential rotation, and meridional circulation).

Key words: MHD – Sun: magnetic fields – dynamo

1. Introduction

Most dynamo models for the magnetic cycle of the Sun involve two basic processes: (i) the production of the toroidal field component due to the stretching of the poloidal component by differential rotation (the Ω-process), and (ii) the generation of the poloidal field due to the twisting of the toroidal field by cyclonic turbulence or other non-mirror-symmetric velocity fields (the α-process). In the formal framework of mean-field electrodynamics the combination of these processes led to the construction of so-called $\alpha\Omega$ dynamo models, which reproduce essential features of the solar cycle if the parameters are suitably chosen (Moffat 1978, Parker 1979, Krause & Rädler 1980).

Problems like the rapid loss of flux by magnetic buoyancy in the convection zone have led to the conjecture that the dynamo does not operate in the bulk of the convection zone but in a thin layer of convective overshoot at its bottom (for a recent review, see Schmitt 1993). On the other hand, calculations on the rise of buoyant magnetic flux from the overshoot layer suggest that the strength of the toroidal field at the bottom of the convection zone should be of the order of 10^5 G (Choudhuri & Gilman 1987; Choudhuri 1989; Moreno-Insertis 1992; Fan et al. 1993, D'Silva & Choudhuri 1993; Schüssler et al. 1994, Caligari et al. 1995). Since this value is one order of magnitude higher than the equipartition value, it is difficult to understand how convective turbulence could twist such a strong field and produce the conventional α-effect. We therefore study a model by placing the α-effect elsewhere, while a layer of strong differential rotation is assumed at the bottom of the convection zone in accordance with recent helioseismology finding (Schou et al. 1992).

The bipolar active regions on the solar surface appear with systematic tilts with respect to the East-West direction due to the Coriolis force acting during the rise through the convection zone (Wang & Sheeley 1991; D'Silva & Choudhuri 1993). The decay of these tilted bipolar regions produces poloidal magnetic flux (Leighton 1964; Wang et al. 1989 and references therein). We follow Durney (1995) in assuming that the generation of the poloidal field from the toroidal field (which gave rise to the bipolar regions) takes place near the solar surface where the bipolar active regions decay and model this process within the framework of mean-field theory by concentrating the α-effect just below the solar surface.

The sense of the observed tilt corresponds to a positive value of α in the northern hemisphere (Stix 1974), whereas helioseismology observations indicate a positive $\partial\Omega/\partial r$ in the lower latitudes (i.e. below about 45° where the strong toroidal field is presumably generated) at the bottom of the convection zone. If these two separated layers of α and differential rotation are coupled by diffusion, then $\alpha(\partial\Omega/\partial r) > 0$ would lead to a poleward propagation of the dynamo wave (Yoshimura 1975; Parker 1993). We, however, couple the two layers by a meridional circulation which is equatorward at the bottom of the convection zone and poleward at the top. A meridional circulation of this kind is indicated by a number of observations (Andersen 1987, Ulrich et al. 1988, Hathaway 1993, Komm et al. 1993) and is required to model the observed poleward migration of the

Send offprint requests to: M. Schüssler diffuse magnetic fields on the solar surface (Wang et al. 1989; Dikpati & Choudhuri 1994, 1995). The effect of meridional circulation on the dynamo action is a subject which seems to have drawn scant attention in the past (Roberts & Stix 1972). If the timescale of the meridional circulation is shorter than the diffusion timescale, then our model leads to butterfly diagrams in qualitative agreement with obervations, by overcoming the constraint that $\alpha(\partial\Omega/\partial r) > 0$ causes poleward propagation. In this *Letter* we do not aim at a detailed model of the solar cycle. A more systematic study will be carried out in a subsequent paper.

2. The model

Writing the axisymmetric magnetic field as the sum of its poloidal and toroidal components

$$\boldsymbol{B} = \nabla \times [A(r,\theta)\hat{e}_\phi] + B(r,\theta)\hat{e}_\phi \tag{1}$$

in spherical coordinates, the equations for the $\alpha\Omega$ dynamo are

$$\frac{\partial A}{\partial t} + \frac{1}{s}(\boldsymbol{v}_P.\nabla)(sA) = \eta\left(\nabla^2 - \frac{1}{s^2}\right)A + \alpha B, \tag{2}$$

$$\frac{\partial B}{\partial t} + \frac{1}{r}\left[\frac{\partial}{\partial r}(rv_r B) + \frac{\partial}{\partial\theta}(v_\theta B)\right] = \\ = \eta\left(\nabla^2 - \frac{1}{s^2}\right)B + s(\boldsymbol{B}_P.\nabla)\Omega, \tag{3}$$

where $s = r\sin\theta$, $\boldsymbol{B}_P = \nabla \times [A\hat{e}_\phi]$ is the poloidal magnetic field, $\boldsymbol{v}_P = v_r\hat{e}_r + v_\theta\hat{e}_\theta$ is the meridional circulation and Ω is the angular velocity.

We solve Eqs. (2) and (3) within a northern quadrant of the convection zone (i.e. within $0.7R_\odot \le r \le R_\odot, 0 \le \theta \le \pi/2$) with the same meridional circulation $\boldsymbol{v}_P$ as used by Dikpati and Choudhuri (1995). We use the value $v_0 = 7$ m s^{-1} for the amplitude of the meridional circulation, which gave the best agreement with the observed evolution of the poloidal surface field in the simulations of Dikpati & Choudhuri (1994, 1995). The α coefficient is taken to be of the form

$$\alpha = \alpha_0\cos\theta.\frac{1}{4}\left[1 + \mathrm{erf}\left(\frac{r - r_1}{d_1}\right)\right]\left[1 - \mathrm{erf}\left(\frac{r - r_2}{d_2}\right)\right] \tag{4}$$

with $r_1 = 0.95R_\odot$, $r_2 = R_\odot$ and $d_1 = d_2 = 0.25R_\odot$ so that α is concentrated in the top layer $0.95 < r/R_\odot < 1.0$. The angular velocity distribution is given by

$$\Omega = \Omega_0\{0.9294 + 0.0353\left[1 + \mathrm{erf}\left(\frac{r - r_3}{d_3}\right)\right]\}, \tag{5}$$

where Ω_0 is the surface rotation rate, and we take $r_3 = 0.7R_\odot$, $d_3 = 0.1R_\odot$ so that the differential rotation is concentrated in a layer of thickness $0.1R_\odot$ at the bottom of the convection zone, with $\partial\Omega/\partial r$ positive. This latitude-independent angular velocity roughly corresponds to the helioseismologically determined rotation profile near the solar equator. Calculations with a more realistic latitude-dependent angular velocity distribution are currently under way and will be presented in a future paper.

A rather low value of turbulent diffusion $\eta = 1.1 \times 10^{11}$ cm$^2\cdot$s^{-1} is chosen in order to let the field advection by the meridional flow dominate over the diffusive transport.

We use the simple boundary conditions for B that it is zero on the three boundaries other than the bottom boundary, where

$$\frac{\partial}{\partial r}(rB) = 0,$$

corresponding to a perfectly conducting solar core. The bottom boundary condition for the vector potential is $A = 0$, whereas the boundary conditions for A on the other three boundaries remain the same as in Dikpati & Choudhuri (1995).

The calculations are performed on a 64×64 grid with a code developed by extending the original code of Dikpati & Choudhuri (1994, 1995), which solved Eq. (2) alone. The differencing scheme and other aspects of the original code have been described in detail by Dikpati & Choudhuri (1994). The code has been extended by using the same differencing scheme for handling Eq. (3). The results presented in the next section are obtained by using $\alpha_0 = 3$ m s^{-1} with α-quenching included. In other words, α_0 decreases in regions where the amplitude of the magnetic field becomes large. This α_0-quenching enables an arbitrary initial state to relax to a periodic solution.

3. Results

Figs. 1 and 2 show butterfly diagrams resulting from calculations without and with meridional circulation. The diagrams give lines of constant strength of the toroidal field on a latitude-time plane, determined near the bottom of the convection zone. Consequently, they represent the toroidal field generated in the shear layer, from where it erupts to the surface and forms active regions.

Fig. 1 is for the case without meridional circulation, so that the two induction layers are coupled purely by diffusion. As expected from the sign combination of the induction effects, the resulting dynamo waves propagate poleward. The oscillation period of about 300 years is determined by the diffusion time. The situation changes drastically if we allow for meridional circulation. As can be seen in Fig. 2, the toroidal field belts now propagate equatorward and the field strength is enhanced in low latitudes, both due to the meridional transport of magnetic flux by the circulation flow. The period has decreased by more than a factor of 3. Near the pole, the toroidal field seems to vary in antiphase with respect to the propagating dynamo wave in the lower latitudes. As can be seen in the field plots in Fig.3, this behaviour is caused by a phase of rapid propagation of the corresponding belt of toroidal field in the course of the poloidal field reversal and the generation of opposite polarity toroidal field near the pole.

Fig. 3 shows a sequence of snapshots of the magnetic field configuration covering one half period. The time difference between subsequent figures is 1/8 of the dynamo period, T. The equatorward propagation of the toroidal field belts in the lower part of the convection zone is clearly visible. The evolution of

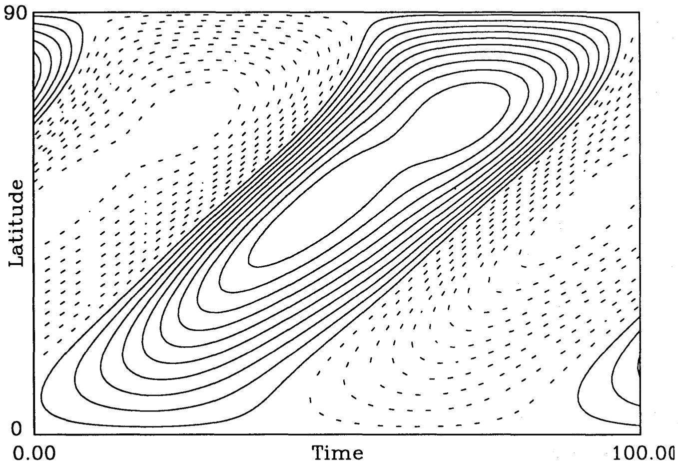

Fig. 1. Butterfly diagram for the toroidal field, taken near the bottom of the convection zone. Meridional circulation is switched off. Time is given in units of $t_0 \simeq 3.5$ years.

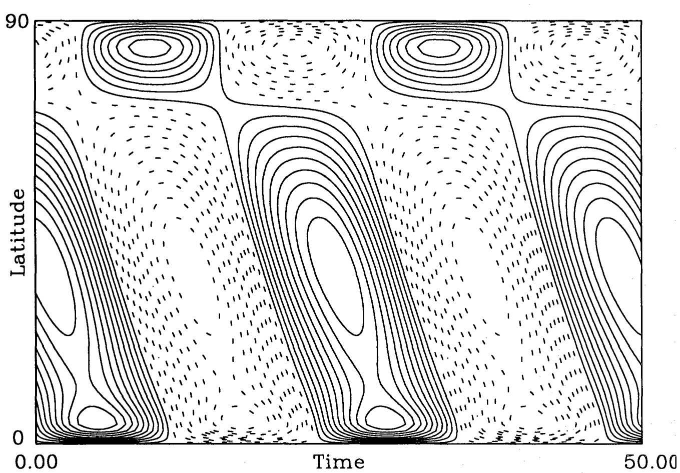

Fig. 2. Same as Fig. 1, but for the case with meridional circulation (v_0 = 7 m$\cdot$s^{-1}). The dynamo waves now propagate equatorward and the period has decreased (note the different time interval).

the poloidal field is more non-uniform; as the opposite polarity field is advected towards higher latitudes, the polar field is rapidly reversed. A new belt of toroidal field is created, which starts to move equatorward. The poloidal field, on the other hand, is concentrated in the polar regions and varies camparatively little until the next reversal.

The equatorward migration of the toroidal field and the poleward migration of the poloidal field are in accordance with the observations that the sunspot belts (resulting from the toroidal field) migrate equatorward, whereas the background magnetic fields on the solar surface (presumably coming from the poloidal component) migrate poleward (see Wang et al. 1989). Previously Dikpati & Choudhuri (1994, 1995) modelled these observations by taking a running dynamo wave as the bottom boundary condition. The present model in contrast is self-consistent, since we solve the dynamo equation.

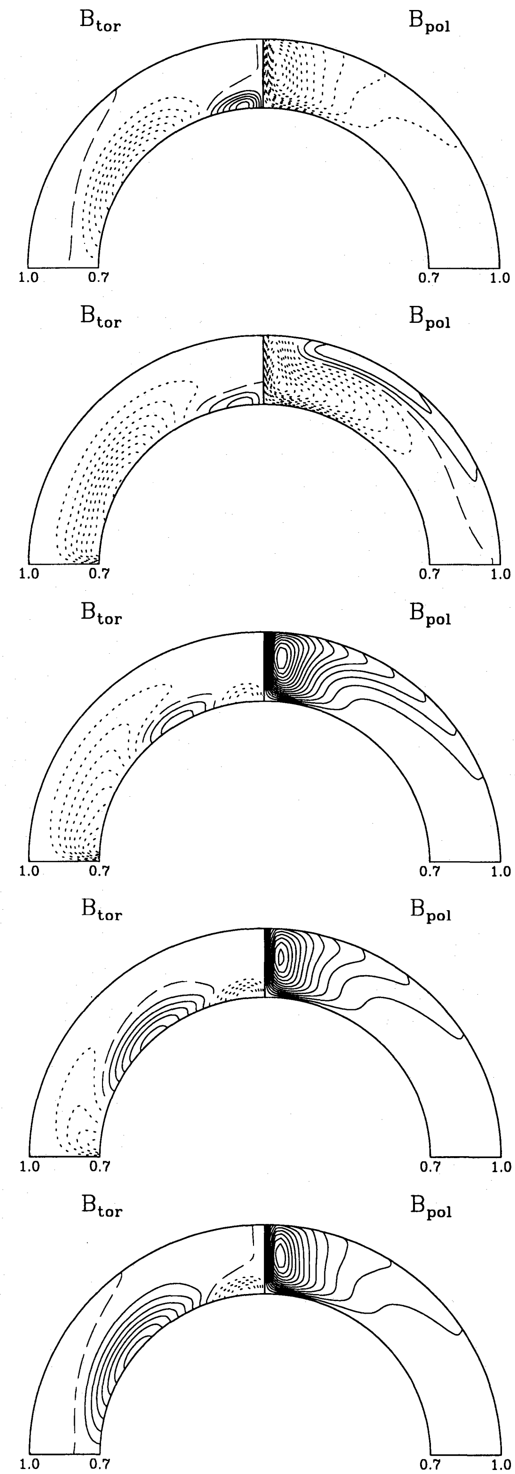

Fig. 3. Time evolution of field configurations in a meridional cut of the convection zone. The panels cover a half period (T) of the calculation corresponding to Fig. 2. From top to bottom: Δt = 0; $T/8$; $T/4$; $3T/8$; $T/2$. Contour lines of the toroidal field are given on the left-hand side, poloidal field lines are drawn on the right-hand side of the panels. Negative polarity is indicated by short-dashed lines; zero lines are denoted by long dashes.

4. Conclusion

We have shown that meridional circulation can lead to solar-type butterfly diagrams with equatorward motion of the activity zone, even if the sign combination of the induction effects would cause a poleward propagation of the dynamo wave. This opens the possibility for a dynamo model directly based on observational data: an α-effect inferred from the observed tilt angles of sunspot groups and the angular velocity distribution as given by recent results of helioseismology.

We have presented here an illustrative example in order to demonstrate the basic validity of this idea. The simple latitude-independent angular velocity profile only near the equator corresponds to solar differential rotation as inferred from helioseismology; hence, this model cannot be directly compared with solar observations. A more detailed study with the full differential rotation law is under way. First results from such calculations show that the basic effect of meridional circulation shown here remains: solar-type butterfly diagrams with equatorward propagation of a few toroidal field belts are produced.

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